Abstract:

This paper revisits the original Allingham and Sandmo (1972) framework with a view towards addressing the issue of tax compliance, and examining the political economy implications of tax evasion for progressivity in the tax structure. In so doing, we 'start from scratch' by constructing a simple extension of the basic Allingham and Sandmo construct that allows agents to initially decide whether to evade taxes or not. We then use a step-by-step model building procedure by taking both the basic model and its 'evade-or-not' counterpart towards a dynamic macroeconomic framework. We find that the 'evade or not' assumption has strikingly different and more realistic implications for the extent of evasion, and demonstrate that it is a more appropriate modeling strategy in the context of macroeconomic models. Furthermore, our numerical analysis suggests that the political outcome for the tax rate for a given level of inequality is conditional on whether there is a large or small or large extent of evasion in the economy, although *changes* in inequality do not matter for this outcome.
Tax Evasion, Inequality and Progressive Taxes: A Political Economy Perspective

Radhika Lahiri\textsuperscript{a} and Mark Phoon\textsuperscript{b}

December 2012

Abstract

This paper revisits the original Allingham and Sandmo (1972) framework with a view towards addressing the issue of tax compliance, and examining the political economy implications of tax evasion for progressivity in the tax structure. In so doing, we ‘start from scratch’ by constructing a simple extension of the basic Allingham and Sandmo construct that allows agents to initially decide whether to evade taxes or not. We then use a step-by-step model building procedure by taking both the basic model and its ‘evade-or-not’ counterpart towards a dynamic macroeconomic framework. We find that the ‘evade or not’ assumption has strikingly different and more realistic implications for the extent of evasion, and demonstrate that it is a more appropriate modeling strategy in the context of macroeconomic models. Furthermore, our numerical analysis suggests that the political outcome for the tax rate for a given level of inequality is conditional on whether there is a large or small or large extent of evasion in the economy, although changes in inequality do not matter for this outcome.

JEL classification numbers: H26; D63; E60
Keywords: Tax Evasion; Inequality; Political Economy

\textsuperscript{a} Queensland University of Technology, Brisbane, Australia.
\textsuperscript{b} British University Vietnam, Hanoi, Vietnam.
1. Introduction

The standard approach to tax compliance applies the economics-of-crime methodology pioneered by Becker (1968); in its first application, due to Allingham and Sandmo (1972), it models the behaviour of agents as a decision involving the choice of the extent of their income to report to tax authorities, given a certain institutional environment, represented by parameters such as the probability of detection and penalties in the event the agent is caught. This issue, however, has not been fully explored in a macroeconomic context. In a macroeconomic context, agents make decisions about many goods and in different time periods, and therefore face many intertemporal and intratemporal trade-offs that are intrinsically linked to the tax evasion decision. For example, the decision to evade will have an impact on the choice of consumption not only on this date, but also on a future date in time. It is then reasonable to speculate that the state contingent planning, and forward looking behaviour typical of macroeconomic models could yield different results.

This paper pursues the above mentioned issues and examines whether these trade-offs are indeed relevant. Our approach is to ‘start from scratch’ by revisiting the very first conceptual formalisation of the tax evasion problem of Allingham and Sandmo (1972) (henceforth, AS), and then use a step-wise model building procedure that extends this model towards a dynamic macroeconomic framework. In doing so, we unearth issues of significant relevance to the macroeconomic modeling of the tax evasion problem. Furthermore, we find that the macroeconomic perspective of the problem, in conjunction with a simple extension of the basic AS construct alleviates some anomalies that are intrinsic to that framework.

To elaborate on this point, we briefly discuss the basic AS model here. In an elegant model using the ‘economics of crime’ idea pioneered by Becker (1968), Allingham and Sandmo model the tax evasion problem as a portfolio decision in which agents choose the amount of income to report to the tax authorities in the presence of uncertainty. The uncertainty stems from the fact that the agents may be audited and have to pay a fine proportional to the underreported amount of income in the event that they are caught.

This model has spawned a large amount of literature aimed at addressing a key limitation of this particular construct.1 In essence, a critical issue pointed out in Sandmo (2005) and previous literature is that the AS model takes a cynical view of the evasion decision - it assumes that the taxpayer does in fact engage in tax evasion given a restriction on the parameters of the model. The question then, is that is this a reasonable assumption?

---

1 See, for instance, Cowell and Gordon 1989; Gordon 1989; Myles and Naylor 1996.
According to Sandmo (p 649, 2005) “While it is difficult to ascertain the exact number of people who evade taxes it is clear that there are several who don’t even though they have the opportunity to do so.”

We first seek to explore this point made by Sandmo by examining a specific parameterisation of the model. This initial exploration is essential from our perspective – we want to take the reader through every step of the model-building process that follows without interruptions due to the need to refer back frequently to the original model. This also helps to motivate each stage of the modeling that follows with greater clarity than would otherwise have been possible. Secondly, our objective is to develop a deeper intuition regarding various aspects that is difficult to obtain in situations where the theoretical analysis provides ambiguous results. For instance, the impact of the tax rate on the proportion of income that is reported is known to be ambiguous (see Allingham and Sandmo 1972, p 329-330). Within the context of our research, however, which examines the link between tax evasion and progressivity, it is important to get an idea of how tax evasion changes as progressivity increases. Furthermore, it is important to do so within the context of a parameterisation of preferences that is commonly used in macroeconomic models. We therefore restrict our analysis to the log utility case, which in turn is a special case of the constant-relative-risk-aversion (CRRA) style of preferences that are typical in macroeconomic models.

Some key insights emerge from this analysis. Firstly, while our results are simply ‘illustrations’ of the more general theory, they provide information that is of value in the parameterisation and development of a macroeconomic model of tax evasion. Specifically, we find that the basic AS model ‘works’ for a very narrow and unrealistic range of parameters, thereby reinforcing and further clarifying the point made by Sandmo (2005). Secondly, since we use a CRRA assumption, we find, as theoretical work in the AS paper suggests, that the proportion of income reported is invariant to their income or wealth. Behavioural research and empirical evidence, however, suggests that proportion of (reported/unreported) income should (decrease/increase) as income increases (see for example, Bloomquist 2003 and references therein). This is a somewhat problematic issue since, to achieve this relationship in the standard model, we would need a decreasing-relative-risk-aversion (DRRA) assumption for preferences. Macroeconomic models, on the other hand, are restricted to use the CRRA assumption in order to produce results that are consistent with steady state growth and certain business cycle features. (See Cooley and Prescott, 1996). This feature of the model motivates some of the extensions of the basic model that are to follow in this paper.
In a subsequent section, we introduce a ‘cost of evasion’ function similar to that of Chen (2003), and show that it is possible to generate the desired relationship between the extent of evasion and income while keeping the CRRA assumption intact. While a ‘fixed cost’ would invariably reduce the extent of evasion, it is an unappealing way of dealing with the problems discussed above, as one is effectively introducing a non-convexity and ‘forcing the issue’, so to speak. On the other hand, however, it is of interest to ask whether an ‘evade or not’ decision can lead to different outcomes without the introduction of a fixed cost.

A key contribution of this paper results from asking this question. Essentially, this involves an agent comparing his/her utility in the case of certainty – i.e. when he/she does not evade with the utility in the case in which he/she undertakes evasion. We find that while this construct does not make a difference in the case of the basic AS model, it makes a substantial difference in the case of other simple extensions of the basic model that are considered in this study. Specifically, when we extend the basic model to include consumption across two time periods, we find that there is a range of parameters for which an agent chooses not to evade. In the extension presented in Section 4, when we introduce a distribution of income with heterogeneous agents, this feature of the model manifests itself in the form of a substantial number of agents choosing not to evade for a reasonably realistic range of values of the tax rate. We emphasise that this result emerges without the introduction of a fixed cost.

Our intuition for this result is as follows: A key feature of macroeconomic models is the desire to smooth consumption over time and across states, and also across different goods. This is a result based on the ‘convexity’ assumption in relation to preferences, which causes ‘balance’ in consumption to be desirable. Now this assumption is present in the basic AS model as well. However in the case of macroeconomic models, the dimensions along which consumption smoothing takes place are more varied relative to microeconomic models. The two-period model is an extension which introduces consumption smoothing along a time dimension. Tax evasion, on the other hand, within a state contingent framework of the type we consider acts contrary to consumption smoothing. In a state contingent model, the agent has to choose a consumption plan which specifies how much to consume in the ‘good’ state in which he/she is ‘not caught’, and how much to consume in the ‘bad’ state in which evasion is detected. The more he/she evades, the more is the disparity across consumption in

2 In our model, and in Chen (2003), the ‘cost of evasion’ is a function of the extent of evasion. We interpret our cost of evasion function to consist of both pecuniary and non-pecuniary elements. The pecuniary component may consist of bribes, while the non-pecuniary element may consist of a sense of guilt from non-payment of taxes.
different time periods and across states. This feature of a macroeconomic model, combined with consumption smoothing will then enhance the desire to not evade.

Finally, we introduce political economy considerations by incorporating wealth heterogeneity among agents, and a lump sum redistributive transfer. We first examine the model without a cost-of-evasion function. We find that the extent of evasion increases with inequality, but for a range of values for the tax rate, the ‘evade or not’ model always produces a lower amount of evasion in comparison to the AS version of the model. Furthermore, an interesting outcome emerges in relation to the mix of evaders in the distribution. Typically, for low values of the tax rate, evasion seems to be concentrated at the bottom end of the income distribution, and this tendency is exacerbated when inequality increases. As this feature of the model seems a little counterintuitive, we then incorporate a ‘cost of evasion function’ that is increasing in the proportion of unreported income, which has the effect of reversing this result.

The results of our political economy extensions with a cost-of-evasion feature have the effect, as described above, to switch the identity of the evaders in the distribution – it is now the rich rather than the poor who evade. Other outcomes, however, remain unchanged. While the enforcement parameters such as the probability of detection and penalties for evasion do not alter the overall political economy outcomes, they do cause significant shifts in the preference profiles of the voters, leading to situations in which non-majority outcomes can occur. In these cases we apply the plurality rule and the winning outcome remains unchanged. If one applies a majority runoff procedure between two of the outcomes with the highest votes, however, then an alternative outcome with a low level of progressivity could be chosen. Alternatively, models with lobbies or other complex voting structures, and those which model an equity-efficiency trade-off by incorporating work-effort could produce a diverse set of outcomes. We leave these extensions as a direction for future research.

The remaining sections are organised as follows. Section 2 revisits the basic original Allingham and Sandmo model, and introduces the model with an ‘evade or not’ choice. Section 3 extends the models to two periods with a view towards building a macroeconomic framework. Section 4 incorporates heterogeneous agents and redistributive transfers into the model. In Section 5, we then proceed by looking at the political economy implications of the previous models by allowing the agents to vote on a range of tax rates presented to them. Finally, in Section 6, we introduce a cost-of-evasion function and analyse the political economy implications of the models. Section 7 concludes.
2. AS Model and its ‘Evade or Not’ Counterpart

A. The Basic Allingham and Sandmo Model

We first begin by revisiting the original Allingham and Sandmo (1972) model and studying a simple parameterisation of it. The aim is to use a type of parameterisation of preferences that is common to macroeconomic models, and thereby develop some intuition regarding the conditions in which tax evasion would emerge in such models. Specifically, we explore the point made by Sandmo (2005), discussed in the previous section, by looking at the case in which utility is logarithmic, which in turn is a special case of the constant-relative-risk-aversion (CRRA) style of preferences that are typical in macroeconomic models.

In the original AS model, the agent’s labour supply is taken as a given (this includes the agent’s gross earnings and income gained from capital). The agent makes his/her decision of the amount of income to report or evade at the moment of filling in his/her tax returns. According to Sandmo (2005), this may be an advantage because it leads to clear and reasonably unambiguous hypotheses.

Assuming log utility, an agent’s preferences are given by the following:

\[ E[U] = (1 - p) \ln(W - \theta X) + p \ln(W - \theta X - \pi(W - X)) \]  

where \( p \) is the probability of being caught, \( W \) is actual income or wealth, \( \theta \) is the tax rate, \( \pi \) is the fine paid from evading taxes, and \( X \) is the amount of income reported. Given that log utility is assumed, \( U \) is increasing and concave so that the tax payer is risk averse. Such an agent chooses the amount of income to report, \( X \), to maximise equation (1). Maximising over the choice of \( X \) yields the following first-order condition for an optimum:

\[ \frac{-\theta(1-p)}{W - \theta X} + \frac{p(-\theta + \pi)}{W - \theta X - \pi(W - X)} = 0 \]  

Solving for the optimal \( X \) we get:

\[ X^* = \frac{\pi p - \theta + \pi \theta (1 - p)}{\pi \theta - \theta^2} W \]

where \( X^* \) is the optimal level of income reported to the tax authorities. This implies that for the log utility case, the proportion of income reported is constant. Intuitively, this result seems a little unreasonable. One would expect, for example, to see a smaller proportion of
income reported by the richer agents in comparison with the poorer agents. In the AS framework this can only be achieved by assuming DRRA preferences.

For an interior solution, the following conditions need to be satisfied:

\[ \frac{\partial E(U)}{\partial X} \bigg|_{X=0} = \frac{-\theta(1-p)}{W} + \frac{p(\pi - \theta)}{W - \pi W} > 0 \]  

(4)

and

\[ \frac{\partial E(U)}{\partial X} \bigg|_{X=W} = \frac{-\theta(1-p)}{W(1-\theta)} + \frac{p(\pi - \theta)}{W(1-\theta)} < 0 \]  

(5)

Equation (4) is satisfied iff:

\[ \Rightarrow \frac{\theta(1-\pi)}{(1-\theta)\pi} < p \]  

(6)

And equation (5) is satisfied iff:

\[ \Rightarrow p < \frac{\theta}{\pi} \]  

(7)

which implies the following restriction to ensure an interior solution:

\[ \frac{\theta(1-\pi)}{(1-\theta)\pi} < p < \frac{\theta}{\pi} \]  

(8)

The condition \( \frac{\theta(1-\pi)}{(1-\theta)\pi} < p \) ensures that amount of reported income, \( X \), is always some positive amount. This is what we have termed the lower bound on \( X \). The upper bound on \( X \), given by the condition \( p < \frac{\theta}{\pi} \), ensures that the amount on reported income cannot be greater than the income itself. The implications of the latter condition were discussed in Sandmo (2005), and we briefly reiterate them here. As suggested by Sandmo (2005), if the penalty rate is twice the regular tax rate the condition in question implies that the probability of detection which would be high enough to deter tax evasion is greater than 0.5. This is far in excess of most empirical estimates and raises the question of whether the model depicts a much greater degree of evasion than is observed in the data.

In the case of lower bound, too, there are some implications of an analogous and intuitively unappealing nature. In Figure 1, we plot the lower bound for two cases, one in
which $\pi = 20$ – the case discussed by Sandmo (2005) in relation to the upper bound, and one in which $\pi=(1.5)0$.³

![Figure 1: Lower bound for Probability of Detection on Tax Rate when $\pi=20$ and $\pi=1.50$.](image)

Figure 1: Lower bound for Probability of Detection on Tax Rate when $\pi=20$ and $\pi=1.50$.

Now, when $\pi=20$, if the tax rate is set at $\theta=0.2$, then the probability of detection would have to be set at an implausibly high value of $p=0.4$ to prevent 100% evasion – i.e to prevent the case in which agent chooses to not report any of his income. Likewise, for $\pi=1.50$, if for example, the tax rate is set at $\theta=0.4$, then the probability of detection would have to be $p=0.55$ to satisfy the lower bound condition. However, within the range in question, the basic Allingahm and Sandmo results are quite intuitive. In the comparative-statics presented in their paper, they show that the extent of income reported is increasing in the penalty rate and probability of detection. The effect of the tax rate $\theta$ is, however, ambiguous.

Furthermore, we return to the question asked in the previous section: Given that the parameters for an interior solution hold, is it possible for the agent’s utility in the ‘certainty scenario’, whereby he chooses not to evade any of his income, to be higher than the expected

³ Although we also plot negative values on the y-axis, as probabilities are between 0 and 1 only, only the graph above the 0 on the y axis is meaningful.
utility under evasion? Specifically, would the outcome of the model be any different if we allowed the agent to first decide whether or not to evade, and then if he or she decides to evade, choose the amount to evade. We consider the ‘evade or not’ choice in the model presented below.

**B: The Allingham and Sandmo Model with ‘Evade or Not’ Choice**

In this case, we compare the indirect utility obtained from substituting (3) into (1) with the indirect utility obtained by solving the model below:

\[ u(C^{ne}) \]  
subject to

\[ C^{ne} = W - \theta W \]  

Here variables are analogously defined with ‘ne’ representing ‘not-evading’. Assuming log utility, the indirect utility function for non-evasion (IUFNE) is given by:

\[ \ln(W - \theta W) = \ln(1 - \theta) W \]

Likewise, we can derive the indirect utility function in the Allingham and Sandmo case by substituting for \( X^* \) derived in equation (3) into the utility function (1). Comparing the indirect utilities of the AS model (labeled IUFAS) and the model without evasion (labeled IUFNE) gives the following: \(^4\)

\[ IUFAS \geq IUFNE \text{ iff } \]

\[ (1 - \delta\theta)^{1-p}(1 - \delta\theta - \pi(1 - \delta))^p \geq (1 - \theta)^p(1 - \theta)^{1-p} \]

\[ \Rightarrow (1 - \delta\theta)^{1-p}(1 - \pi + \delta(\pi - \theta))^p \geq (1 - \theta)^{1-p}(1 - \theta)^p \]  

where \( \delta = \left( \frac{\pi(1-p) - \theta}{\pi\theta - \theta^2} \right) \). We can see from equation (12) that the first term on the left-hand side (LHS) is always going to be greater than the first term on the right-hand side (RHS) as \( 0 < \delta < 1 \) if the conditions for an interior solution are satisfied. For the indirect utility in the AS model to always be greater than the indirect utility of the not-evade alternative, we would

\(^4\) For a derivation of this inequality see Appendix A.
also require the second term on the LHS to be greater than the second term on the RHS. Again, we can show that if the conditions for an interior solution are satisfied, the second term on the LHS is less than the second term on the RHS, making it difficult to compare the expressions of both side of the inequality.\(^5\)

While our numerical simulations for this case show that IUFAS > IUFNE for a large range of parameters compatible with condition (8), it is not possible to prove this analytically. However, as will become clear in the subsequent sections, it is easy to disprove an analogous proposition via numerical example in the two-period extension of the basic model, which is considered in the next section. Specifically, in the next model we consider in Section 3, there is a range of parameters for which the interior solutions are satisfied, and the utility under certainty with no evasion is higher. We next consider the results of our numerical experiments of the two models.

**Numerical Experiments:**

The ‘benchmark’ set of parameters of the models in this paper are: \(\theta=0.35; \ p=0.3; \ \pi=\theta+0.32; \ r=0.06.\)\(^6\) We conduct experiments only on the range of parameter values in which the conditions for an interior solution are satisfied.\(^7\) For the purpose of sections 4 onwards we also assume a wealth level \(W=25.\)

We first consider how, in the basic AS model, the extent of evasion varies with the tax rate \(\theta,\) a feature that will be of relevance when interpreting the results of the extensions that follow.\(^8\) Figure 2 shows that the relationship between the proportion of unreported income (\(\alpha\)) and the tax rate (\(\theta\)) is non-monotonic. The proportion of unreported income is increasing for lower levels of the tax rate but decreasing for relatively higher levels. The highest proportion of unreported income, where \(\alpha=0.88,\) occurs at a tax rate of around \(\theta=0.4.\) A possible interpretation for Figure 2 could be due to income and substitution effects associated with the changes in the tax rate. The substitution effect captures the fact that as tax rates rise, the opportunity cost from not evading becomes higher. The ‘income effect’, according to

---

\(^5\) That the second term of the LHS is smaller than the second term of the RHS, provided \(p\pi<\theta,\) is shown in Appendix A.

\(^6\) Our choice of benchmark parameters is also related to the condition for an interior solution – we select them in a way that permits a reasonable range for the simulations presented below. At this stage, given that we are dealing with a fairly simple extension of the basic AS model, a full-fledged ‘calibration’ exercise is not feasible.

\(^7\) When conducting numerical experiments on specific parameters, the rest of the parameters are held at their benchmark rate.

\(^8\) Note that we can write the amount of reported income as \(X = (1 - \alpha)W,\) where \(\alpha\) is the proportion of unreported wealth.
Allingham and Sandmo, should be zero for the case of CRRA preferences. However, there is another aspect of the substitution effect here: as the tax rate goes up and, the amount of expected penalties and fines increase for a given proportion of income evaded, making the opportunity cost of not evading lower. In the figure below, the former effect seems to dominate in the range in which the tax rate increases from 0.15 to 0.4 and the proportion of unreported income increases. This latter effect comes into play when the tax rate increases from 0.4 to 0.65.

![Figure 2: Proportion of Unreported Income as the Tax Rate Increases.](image)

We also illustrate our earlier discussion in relation to an ‘evade or not’ option for the agent by comparing the utility under certainty which is labelled as IUFNE and the utility in the AS model labelled as IUFAS in Figure 3. Figure 3 shows that the indirect utility function of the AS model (represented by the blue line) is always higher than the indirect utility function of the ‘evade or not’ choice (represented by the green line) for the range of parameters that satisfy the interior solution conditions. In this case, therefore, and ‘evade or not’ extension is not applicable. The reason for presenting this comparison at this point is
motivated by the fact that it makes it easier to present and discuss a reversal of this result in subsequent sections.

Figure 3: Comparison of Indirect Utility Functions of AS Model and ‘Evade or Not’ Model.

3. AS Model with Two-Periods and its ‘Evade or Not’ Counterpart

A: The Allingham and Sandmo Two-Period Model

We now focus our attention on the AS model with time dimensions by introducing two periods of consumption in the agent’s utility function. This amounts to grafting the tax evasion decision into a standard Fisher (1930) two period small open economy model of the type analysed by Obstfeld and Rogoff (1996). Assuming log utility, the preferences of an individual are:

\[ U(C_1^c, C_1^{nc}, C_2^c, C_2^{nc}) = \ln C_1^c + \ln C_2^c + (1 - p)\ln C_1^{nc} + (1 - p)\ln C_2^{nc} \]  

(13)

where the superscripts \( c \) and \( nc \) denote the states where the agents are caught and not caught respectively, and the subscripts 1 and 2 denote the different time periods, and the variable \( C \) refers to consumption. Such an individual maximises equation (13) subject to the following
period 1 and 2 budget constraints depending on whether or not his evasion has been detected. Equations (14) and (15) below refer to the budget constraints for an individual in periods 1 and 2 in the state that he is caught. When the state is ‘not caught’ his budget constraints are given by equations (16) and (17).

\[
C_1^c + S_1^c = W - \theta X - \pi(W - X) \quad (14)
\]

\[
C_2^c = (1 + r)S_1^c \quad (15)
\]

\[
C_1^{nc} + S_1^{nc} = W - \theta X \quad (16)
\]

\[
C_2^{nc} = (1 + r)S_1^{nc} \quad (17)
\]

As is obvious from the equations above, the agent has no wealth endowment in the second period of his life and therefore must save to finance consumption in period 2. We assume a small open economy in our model where \( r \) is the world interest rate that is taken as a given by the agent. In addition, consumption and saving (denoted \( S \)) in the first period must not exceed his or her disposable wealth, which further depends on whether his or evasion has been detected. In this case, if we substitute (14)-(17) into (13), the first order conditions for \( S_1^c, S_1^{nc} \) and \( X \) are given by:

\[
-\frac{p}{C_1^c} + \frac{p(1 + r)}{C_2^c} = 0 \Rightarrow C_2^c = (1 + r)C_1^c \quad (18)
\]

\[
-\frac{1 - p}{C_1^{nc}} + \frac{(1 - p)(1 + r)}{C_2^{nc}} = 0 \Rightarrow C_2^{nc} = (1 + r)C_1^{nc} \quad (19)
\]

\[
\frac{(\pi - \theta)p}{C_1^c} = \frac{\theta (1 - p)}{C_1^{nc}} \Rightarrow C_1^c = \frac{(\pi - \theta)p}{\theta (1 - p)} C_1^{nc} \quad (20)
\]

It is then straightforward to manipulate equations (14)–(20) to express the variables \( S_1^c, S_1^{nc}, C_1^c, C_1^{nc}, C_2^c, \) and \( C_2^{nc} \) in terms of \( W \) and \( X \):

\[
C_1^c = \frac{1}{2} [W - \theta X - \pi(W - X)] = S_1^c \quad (21)
\]

\[
C_2^c = \frac{(1 + r)}{2} [W - \theta X - \pi(W - X)] \quad (22)
\]

\[
C_1^{nc} = \frac{1}{2} [W - \theta X] = S_1^{nc} \quad (23)
\]

\[
C_2^{nc} = \frac{(1 + r)}{2} [W - \theta X] \quad (24)
\]
Substituting for $C_1^c, C_1^{ne}$ into (14) we can solve for $X$ as follows:

$$
\frac{1}{2} \left[ W - \theta X - \pi (W - X) \right] = \frac{(\pi - \theta)p}{\theta(1 - p)}
$$

Solving for the equation above for $X$, we get:

$$
X^* = \frac{\pi p - \theta + \pi \theta(1 - p)}{\pi \theta - \theta^2} W
$$

We can see that the proportion of reported income, $X^*$, is identical to the AS model, as evident by comparing equation (3) with equation (25). This implies that extending AS model by incorporating two periods and state-contingent planning of consumption and savings does not have any bearing on the proportion of income evaded, or the comparative static analyses of how this proportion of income changes with respect to the parameters $\pi$, $\theta$ or $p$.\(^9\) Likewise, it is easy to show that the restriction for an interior solution is the same as equation (8).\(^{10}\)

However, we will shortly find that it has an interesting implication for the ‘evade or not’ choice discussed earlier.

\textit{B: The Allingham and Sandmo Two-Period Model with ‘Evade or Not’ Choice}

Once again, we can compare the indirect utility obtained from equation (14) with the indirect utility obtained by solving the model below:

$$
\begin{align*}
&u(C_1^{ne}) + u(C_2^{ne}) \\
\text{subject to} \quad &C_1^{ne} + S_1^{ne} = W - \theta W \quad (27) \\
& C_2^{ne} = (1 + r)S_1^{ne} \quad (28)
\end{align*}
$$

Here variables are analogously defined with ‘ne’ representing ‘not-evading’. Assuming log utility, and deriving the optimal consumption and saving plans we can substitute them into (26) to derive the indirect utility function for non-evasion (IUFNE), which is given by:

---

\(^9\) As noted above, these analyses have been performed in the seminal Allingham and Sandmo (1972) paper, for a more general case of the utility function, so such a repetition is unnecessary here.

\(^{10}\) Again such a derivation is unnecessary given our extension has yielded the same expression for $X^*$ that the basis AS model did, but for the sake of completeness a derivation of the conditions for an interior solution in this particular case is presented in Appendix B.
\[
\ln(1 + r) + \ln(1 - \theta)W
\]

Comparing the indirect utilities of the AS model (IUFAS) and the ‘evade or not’ model (IUFNE) it can be shown that: \(^{11}\)

\[
\text{IUFAS} \succeq \text{IUFNE} \iff 2\ln \left(\frac{1 - \theta X^*}{W}\right) + 2p\ln(1 - \pi) \geq 2\ln \left(\frac{1 - \theta}{2}\right) \tag{30}
\]

where \(X^*\) is given by equation (25).

Comparing the utilities of the two models, it is again not possible to prove the proposition that the utility with evasion (IUFAS) is higher than the certainty scenario (IUFNE) given that the conditions for an interior solution are satisfied. However, in this instance, we are able to disprove this proposition via numerical example. Specifically, there is a range of parameters for which the interior solutions are satisfied, and the utility under certainty with no evasion is higher. This result suggests that the ‘evade or not’ formulation is the more appropriate construct in the two-period context, and are illustrated by the following numerical experiments below. \(^{12}\)

**Numerical Experiments:**

From Figure 4, we can see that, in this two-period model, the indirect utility function of the AS model (IUFAS) is not higher than the indirect utility function for the ‘evade or not’ model (IUFNE) for a range of values of the tax rate that are consistent with restrictions for an interior solution for \(X\), the amount of reported income. For relatively low tax rates, i.e. for \(\theta\) between 0.15-0.25, we find that choosing not to evade gives the agents a higher utility. In this case therefore, an appropriate modelling of the tax evasion decision should incorporate an ‘evade or not’ choice. In what follows, we build on this model further by introducing heterogeneous agents and redistributive transfers. However, for the sake comparison, we also present corresponding outcomes in analogous versions of the AS economy, in which agents do not have an ‘evade or not’ choice.

---

\(^{11}\) See Appendix C for this derivation.

\(^{12}\) We do not present the comparative statics analysis showing how \(X\) varies with the other parameters \(p, \pi, \) and \(\theta\) as the outcomes in that respect are identical to those of the basic model.
4. Two-Period Model with Heterogeneous Agents and Redistributive Transfers

A: The Allingham and Sandmo Two-Period Model Heterogeneous Agents and Redistributive Transfers

We now take a step towards developing the model so that political economy aspects may be addressed. This involves the construction of a model with heterogeneous agents, so that the distributional implications of taxes and tax evasion may be considered. The political economy angle is then modelled in a simple way by allowing the agents to vote on their desired tax structure. The agent’s decision making process now involves redistributive transfers. It is assumed that the tax-authority maintains a balanced budget so that average revenue collection is the lump-sum transfer given to all individuals. Individuals do not pay taxes, or receive transfers in the second period of their lives. The expected revenue for lump-sum transfers in the AS model is given by:

\[
\sum_{i=1}^{N} \theta X_i + p \sum_{i=1}^{N} \{\pi(W_i - \theta X_i)\}
\]  

(31)
In the above equation, the first term is total revenue collected for income reported while the second term is the expected revenue of the fines collected from agents who evade taxes and are caught, where the expected revenue collected from these agents is the total revenue evaded multiplied by the probability of detection $p$.\textsuperscript{13} The preferences of an individual are the same as that in equation (13) but the budget constraints are altered and given by the following:

\[
C_1^c + S_1^c = W + TR - \theta X - \pi(W - X) \quad (32)
\]

\[
C_2^c = (1 + r)S_1^c \quad (33)
\]

\[
C_1^{nc} + S_1^{nc} = W + TR - \theta X \quad (34)
\]

\[
C_2^{nc} = (1 + r)S_1^{nc} \quad (35)
\]

where $TR$ represents redistributive transfers, which are computed by averaging the total revenue collected in expression (31) over all agents in the economy. We can see that the budget constraints are now different from previous models as a result of the inclusion of redistributive transfers. The disposable income of the agent has now increased which would alter his/her consumption and saving plans, and his or her desired tax rate.\textsuperscript{14} One can also anticipate further differences to emerge in the ‘evade or not’ formulation, given that transfers in a model with an ‘evade or not’ choice would have to be calculated differently.

Note that the introduction of a lump sum transfer will not have an impact on the first order conditions of the agent’s optimisation problem, and identical steps are involved in deriving the optimal plans, which is now given by:

\[
C_1^c = \frac{1}{2}[W - \theta X - \pi(W - X) + TR] = S_1^c \quad (21')
\]

\[
C_2^c = \frac{(1 + r)}{2}[W - \theta X - \pi(W - X) + TR] \quad (22')
\]

\[
C_1^{nc} = \frac{1}{2}[W - \theta X + TR] = S_1^{nc} \quad (23')
\]

\[
C_2^{nc} = \frac{(1 + r)}{2}[W - \theta X + TR] \quad (24')
\]

In addition, the proportion of reported income, $X$, is also identical to the expression derived for the basic AS model (see equation (3)).

\textsuperscript{13} For large $N$ it is not unreasonable to assume that the probability of detection $p$ is also the proportion of agents in the economy who have been detected evading their incomes.

\textsuperscript{14} Note that transfers are taken as given by the agent, in the sense that he or she cannot individually influence the vote on taxes.
B: The ‘Evade or Not’ Choice Model with Heterogeneous Agents and Redistributive Transfers

Again the ‘evade or not’ model involves a comparison analogous to that discussed in Section 3 above. Note, however, that there is a more complex calculation involved for the agent in the ‘evade or not’ economy in relation to redistributive transfers. In the ‘evade or not’ choice economy, the revenue collected for redistributive transfers is a combination of two parts. The first part is the revenue collected from agents who do not evade and pay taxes on their actual income. The second part involves the revenue collected from the taxes paid by agents who evade, and the fines collected from the agents who evade and are caught. The revenue collected from agents who do not evade \( T_{NE} \) is given by the following:

\[
T_{NE} = \sum_{i \in E} \theta W_i
\]  
(36)

The expected revenue collected from agents who evade taxes \( T_E \) is given by the following:

\[
T_E = \sum_{i \in E} \theta X_i + p \sum_{i \in E} \{\pi(W_i - \theta X_i)\}
\]  
(37)

The total revenue collected in the ‘evade or not’ model is then the sum of \( T_{NE} \) and \( T_E \). This revenue is averaged over all agents and distributed as a lump sum transfer \( TR \).

Once the transfer for the ‘evade or not’ model is computed, it is easy to describe the optimisation problem. Those who do not evade solve an analogous version of the certainty problem in Section 3 B, with the transfer term appearing on the RHS of the period 1 budget constraint. Those who evade face a problem analogous to the one described in Section 3 A, with the term for transfers appearing in counterparts of equations (21)-(24). An agent decides whether or not to evade by comparing the respective utilities. Again, we cannot make an analytical comparison of the utility functions and have to resort to numerical simulations, which are reported in the next section. Also note that for agents who evade the amount of evasion is still determined by equation (3).

We have so far not described the political economy aspect of this model. Before we do so, it is instructive to gain some intuition and insight regarding the introduction of

---

15 Again not that agent’s regard this as exogenous as they cannot individually affect the aggregate transfer in the economy. However, there is full and perfect information in this environment – each agent knows the distribution of wealth and the preferences of the other agents and is therefore able to compute the size of this transfer.
redistribution by means of some numerical simulations. These simulations are presented below.

**Numerical Experiments for Inequality**

We first start by presenting numerical experiments on the number of evaders in the economy when income inequality varies for different levels of the tax rate. Table 1 below illustrates these results.

Table 1: Number of Evaders for Different Levels of Inequality

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$Gini=0.2735$</th>
<th>$Gini=0.3439$</th>
<th>$Gini=0.3807$</th>
<th>$Gini=0.4073$</th>
<th>$Gini=0.5895$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Evaders</td>
<td>Evade or Not</td>
<td>No. of Evaders</td>
<td>Evade or Not</td>
<td>No. of Evaders</td>
<td>Evade or Not</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>17</td>
<td>189</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.30</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.35</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.40</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.45</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.50</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.55</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$Gini=0.5975$</th>
<th>$Gini=0.6736$</th>
<th>$Gini=0.7143$</th>
<th>$Gini=0.8346$</th>
<th>$Gini=0.8388$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Evaders</td>
<td>Evade or Not</td>
<td>No. of Evaders</td>
<td>Evade or Not</td>
<td>No. of Evaders</td>
<td>Evade or Not</td>
</tr>
<tr>
<td>0.15</td>
<td>262</td>
<td>303</td>
<td>305</td>
<td>389</td>
<td>393</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>282</td>
<td>299</td>
</tr>
<tr>
<td>0.25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
For the range of values below $\theta = 0.3$, we note the following.\(^{16}\)

(a) We can see that different levels of inequality give rise to different outcomes for tax evasion; for the range of parameters considered here, the number of evaders increase as inequality increases. When $\theta = 0.15$ for example, the number of evaders increases from 8 to 17 when inequality rises from 0.3807 to 0.4073, and increases further to 189 when the Gini-coefficient is raised again to 0.5895. This means that the number of non-evaders in this economy is around 62.28%. This is consistent to the results found in Christian (1994) where 60% of U.S taxpayers do not understate their income.

(b) For a given level of inequality, the relationship between the number of evaders and tax rates is a little less clear. From the results there seems to be a non-monotonic relationship between the number of evaders and the tax rates. For example, when the Gini-coefficient is at 0.4073, the number of evaders tallies at 17 for $\theta = 0.15$. When $\theta$ rises to 0.20 and 0.25, however, the economy has no evaders. For all other values of $\theta$, ie. 0.30-0.55, the model is similar to the AS model where all agents in the economy evade to some extent from the payment of taxes. This non-monotonicity is hard to explain, but intuition suggests that the non-monotonic relationship between tax rates and the proportion of unreported income, discussed earlier, may have something to do with it.

Specifically, we saw in Figure 2, once the tax rate increases to beyond $\theta = 0.4$, the proportion of unreported income decreases as $\theta$ increases. This means that while all agents in the economy are evading, they evade

\(^{16}\) Note that the ‘evade or not’ economy is identical to the AS economy for values of the tax rate greater that equal to 0.30.
smaller proportions of their income as the tax rate increases beyond 0.4. Within this range, the choice is between not evading at all or evading a relatively small proportion of their income. The ‘evade or not’ choice prior to that point, however, entails a comparison of utility from ‘not evading’ with evading a proportion of income that increases as $\theta$ increases. This proportion increases steeply for low values of $\theta$, before the substitution effect of expected penalties and fines associated with higher tax rates makes the evasion decision more costly. These differences in the two ranges of $\theta$ could be the reason underlying the results we see in Table 1.

In addition, an interesting feature of this model relates to the identity of the evaders in the ‘evade or not’ variant; it is the poorest agents in this model that engage in tax evasion. The benefit from evading in this model is more pronounced at the lower end of the income distribution, as the consumption gains from evading are higher when the level of consumption is low. This could, in part, also explain why the number of evaders increases as inequality increases in the first two rows of Table 1; as inequality increases, there is a greater mass of agents at the lower end of the income distribution, and all these agents experience higher marginal consumption gains from evasion.

5. Political Economy Extensions of Two-Period Model with Heterogeneous Agents and Redistributive Transfers without Cost-of-Evasion

Here we consider extensions of the models presented in Section 4 above to include a political economy determination of one of the parameters of the tax system. Essentially, we assume that voting takes place at the beginning of the period and agents are allowed to vote on $\theta$. After the vote, in economy A (the AS model) agents make their evasion decision and state contingent plans, followed by the auditing by tax authorities, after which transfers are made and the state contingent plans are carried out. In economy B (‘evade or not’ model), the only difference is that after the vote agents decide whether or not to evade, and if they choose to evade, they decide how much to evade. Subsequently, auditing takes place, transfers are made, and consumption and saving plans are carried out. The timing of events of the political economy versions of the two economies is described in Figures 5 and 6 below.
Figure 5: Timeline for the basic model.

Figure 6: Timeline for model with ‘evade or not’ choice.
Numerical Experiments:

We now present some numerical results of the two models regarding the political economy outcomes. Before we do so, however, it is instructive to look at the indirect utility as a function of the tax rate, for various agents at different positions in the income distribution. Figure 7 plots does this for the case of the AS version of the model. Here, Agent 1 represents the poorest agent, while Agent 501 represents the richest agent in the economy, and agents are arranged in ascending order of their income or wealth. Therefore Agent 251, for example, is the median agent in the sample income distribution considered here.

![Figure 7: Agents' preferences over $\theta$ in the AS economy. (Simulation based on a 501 agent economy, indexed in ascending order of their initial wealth, with benchmark parameters).](image)

We can see that the preferences of Agent 1 and Agent 251, over a range of tax rates, are non-single peaked. These agents prefer lower and higher levels of tax rates in relation to some of the ‘in-between’ values. In addition, the results also show that, as expected, agents on the higher end of the income distribution prefer relatively low tax rates whereas, agents on
the low to middle end of the income distribution prefer relatively higher tax rates. Note that the ‘non-single peakedness’ observed in some cases will have interesting implications for the political economy outcome of the vote on $\theta$, the tax rate, as the standard median voter theorem due to Black (1948) no longer applies in this instance. In Figure 8 below, we present the utility functions of the agents in the ‘evade or not’ economy.

![Agent 1](image1.png) ![Agent 251](image2.png) ![Agent 451](image3.png) ![Agent 501](image4.png)

Figure 8: Agents’ preferences over $\theta$ in the ‘Evade or Not’ economy. (Simulation based on a 501 agent economy, indexed in ascending order of their initial wealth, with benchmark parameters).

The utility functions of the agents in the ‘evade or not’ economy differ somewhat slightly than that of the AS model but are still non-single peaked. Furthermore, a greater degree of non-monotonicity in the preferences has been created by incorporating the ‘evade or not’ choice. In this model, agents on the higher end of the income distribution still prefer relatively lower tax rates, and agents on the very low end of the distribution still prefer high tax rates. In the case of the middle income agents, however, the ‘non-single peakedness’ is
more dramatic; they prefer the relatively extreme levels of tax rates to the ‘in-between’ cases, and seem to be indifferent between relatively low tax rates and high tax rates.\textsuperscript{17}

The results of the political economy outcomes are presented in Figures 9 and 10 below. These figures present the percentage of votes in favour over different values of $\theta$, for our benchmark parameters with the Gini coefficient of the distribution set at 0.4073. Figure 9 presents the results for the AS economy while Figure 10 represents the corresponding outcomes for the ‘evade or not’ economy. In relation to voting outcomes of the model, we find that in both the AS model and the model with the ‘evade or not’ choice the winning value of $\theta$ is 0.55, with 72.85\% of the vote in the AS economy and 51.30\% of the vote in the ‘evade or not’ economy. This is the highest choice available to the agents given the interior condition discussed in previous sections. As mentioned before, we find that the political economy outcome of this model is driven by equity considerations, in spite of the presence of tax evasion. This is in part due to the fact that firstly, redistribution occurs even in the presence of evasion, and secondly, there are no administrative costs so that revenue collected from penalties and fines can be used for redistribution. We see that in both models, the highest available tax rate is preferred in the presence of inequality.

What is also interesting is that the distribution of votes in the two models differs on the low-end of the tax spectrum. In the AS economy, the second highest percentage of votes is for the tax rate $\theta=0.2$. In the ‘evade or not’ economy, however, the second highest percentage of votes is for the lowest value of the tax rate available, $\theta=0.15$. Note that in the presence of different voting procedures, such as a majority run-off, the political economy outcomes for the tax rate could be very different. In a majority run-off procedure, for example, there is a second stage to the voting process in which the outcomes with the highest and second-highest votes are pitted against each other. In such cases, if the outcome with the highest number of votes does not have a majority, interesting outcomes can occur depending on how voter preferences are distributed in relation to the remaining two alternatives. Furthermore, given the nature of preferences described in the Figures 7 and 8, it is evident that

\textsuperscript{17} Note that the non-single peakedness of preferences is a feature common to several political economy models with voting, such as Epple and Romano (1996a) and also in political economy models of tax evasion, such as the model described in Borck (2009). Typically, outcomes in these models can be determined by groups of individuals other than the median voter. Sometimes an ‘ends against the middle’ feature appears so that agents at the bottom and top ends of the distribution determine the outcome of the vote. As will be evident later, this can happen within the context of the models discussed in this paper. Even so, within the context of these models it is a relatively rare occurrence and happens in some cases for a narrow range of parameters.
richer models that include lobbies or power groups could lead to a different political outcome, and would consequently be an interesting direction of future research.

Figure 9: Percentage of votes for various values of $\theta$ in the AS economy.
Next we look at the voting outcomes on $\theta$ with various levels of inequality. We can see that in general, increasing the level of inequality does not alter the voting outcome of the tax rate in both models. Only for a very low level of inequality (Gini=0.2735) in the ‘evade or not’ model does the voting outcome change. In this instance, the highest percentage of votes is for the tax rate of $\theta=0.2$. This is a striking difference between the two models. On the other hand, in the AS model, for a low level of inequality (Gini=0.2735), the highest percentage of votes is for the tax rate $\theta=0.55$. This result is perhaps due to the increased non-monotonicity in the agent’s utility function that we saw in Figure 8; such features typically make intuitive interpretations challenging.

It is also interesting, that in the ‘evade or not’ model, for a range of inequalities, the winning vote on $\theta$ is not a majority outcome but a plurality one. If we allowed the voting outcome to be determined by a majority runoff in this case, we would still end up with 0.20 as the winning outcome in the ‘evade or not’ case. This is due to the fact that 29.94% of the votes was for $\theta=0.55$ and the remaining 26.95% of the votes was for a tax rate of $\theta=0.15$. A majority runoff in this scenario would not alter the outcome of the tax rate as those agents who voted for $\theta=0.15$ would now vote for $\theta=0.20$ as opposed to $\theta=0.55$. Referring back to Table 1, recall that for low values of the tax rate, all agents typically chose not to evade. It
would seem, then, that preferences of the majority of agents are to choose a tax rate that leads to less evasion, and in this case a tax rate of $\theta = 0.20$ leads to no evasion in the economy.

Table 2: Vote on $\theta$ for Different Levels of Inequality

<table>
<thead>
<tr>
<th>Gini</th>
<th>Vote on $\theta$ (Evade or Not)</th>
<th>% in Favour (Evade or Not)</th>
<th>Vote on $\theta$ (AS model)</th>
<th>% in Favour (AS model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2735</td>
<td>0.2000</td>
<td>43.1138</td>
<td>0.5500</td>
<td>72.2555</td>
</tr>
<tr>
<td>0.3439</td>
<td>0.5500</td>
<td>40.9182</td>
<td>0.5500</td>
<td>74.2515</td>
</tr>
<tr>
<td>0.3807</td>
<td>0.5500</td>
<td>48.1038</td>
<td>0.5500</td>
<td>71.8563</td>
</tr>
<tr>
<td>0.4073</td>
<td>0.5500</td>
<td>51.2974</td>
<td>0.5500</td>
<td>72.8543</td>
</tr>
<tr>
<td>0.5895</td>
<td>0.5500</td>
<td>67.2655</td>
<td>0.5500</td>
<td>78.0439</td>
</tr>
<tr>
<td>0.5975</td>
<td>0.5500</td>
<td>67.6647</td>
<td>0.5500</td>
<td>77.4451</td>
</tr>
<tr>
<td>0.6736</td>
<td>0.5500</td>
<td>73.0539</td>
<td>0.5500</td>
<td>78.6427</td>
</tr>
<tr>
<td>0.7143</td>
<td>0.5500</td>
<td>77.2455</td>
<td>0.5500</td>
<td>82.0359</td>
</tr>
<tr>
<td>0.8346</td>
<td>0.5500</td>
<td>86.6267</td>
<td>0.5500</td>
<td>89.0220</td>
</tr>
<tr>
<td>0.8388</td>
<td>0.5500</td>
<td>86.0279</td>
<td>0.5500</td>
<td>88.2236</td>
</tr>
</tbody>
</table>

6. Political Economy Extensions of Two-Period Model with Heterogeneous Agents and Redistributive Transfers with Cost-of-Evasion

A: The Allingham and Sandmo Model with Cost of Evasion

Finally, we analyse a two-period political economy model with a cost function associated with evading taxes. The model is similar to the one described in the earlier section with one exception, the decision to evade taxes, as in Chen (2003), it now involves a cost described by $d(\alpha)$, which is increasing in $\alpha$, and where $\alpha$ is the proportion of unreported income. As discussed earlier, we interpret our cost of evasion function to consist of both pecuniary and non-pecuniary elements. The pecuniary component may consist of bribes, while the non-pecuniary element may consist of a sense of guilt from non-payment of taxes. In addition, agents are heterogeneous in their wealth endowment. The preferences of an individual are given by the following:

$$U(C_1^n, C_1^{nc}, C_2^n, C_2^{nc}) = plnC_1^n + plnC_2^n + (1 - p)lnC_1^{nc} + (1 - p)lnC_2^{nc}$$

(38)

such an individual maximises equation (52) subject to the following budget constraints:
\[ C_1^e + S_1^e = W + TR - \theta(1 - \alpha)W - d_0\alpha^2 - \pi(W - (1 - \alpha)W) \]
\[ C_2^e = (1 + r)S_1^e \]
\[ C_1^{ne} + S_1^{ne} = W + TR - \theta(1 - \alpha)W - d_0\alpha^2 \]
\[ C_2^{ne} = (1 + r)S_1^{ne} \]

Here \( TR \) represents transfers, \( X = (1 - \alpha)W \) where \( \alpha \) (the decision variable in this instance) is the amount of unreported income, and \( d_0\alpha^2 \) is the cost function that varies with \( \alpha \). The first-order conditions, with respect to \( S_1^e, S_1^{ne}, \) and \( \alpha \), for an optimum are now given by the following:

\[ \frac{p}{C_1^e} + \frac{p(1 + r)}{C_2^e} = 0 \Rightarrow C_2^e = (1 + r)C_1^e \] (43)

\[ \frac{1 - p}{C_1^{ne}} + \frac{(1 - p)(1 + r)}{C_2^{ne}} = 0 \Rightarrow C_2^{ne} = (1 + r)C_1^{ne} \] (44)

\[ pC_1^{ne}\{(\pi - \theta)W + 2d_0\alpha\} = (1 - p)C_1^e\{\theta W - 2d_0\alpha\} \] (45)

We can see that the first-order condition with respect to \( \alpha \) is no longer the same as in the previous models. In addition, conditions for an interior solution have changed in this case.\(^{18}\)

**B: ‘Evade or Not’ Choice Model with Cost of Evasion**

Again the ‘evade or not’ model involves a comparison analogous to that discussed in Section 3 above. For completeness, however, we state them briefly here. The preferences of an individual in the ‘evade or not’ model are:

\[ \ln(C_1^{ne}) + \ln(C_2^{ne}) \]

Such an individual maximises equation (46) subject to the following budget constraints:

\[ C_1^{ne} + S_1^{ne} = W + TR - \theta W \] (47)

\[ C_2^{ne} = (1 + r)S_1^{ne} \] (48)

Those who do not evade solve an analogous version of the certainty problem in Section 3A, with the budget constraints given by equations (47)-(48). Those who evade face a problem analogous to the one described in Section 3B, with the budget constraints given by equations (39)-(42). Likewise, an agent decides whether or not to evade by comparing the respective

\(^{18}\) We cannot, however, derive these conditions analytically, and we no longer have an analytical solution to the agents’ problem. While we can express all other variables in terms of \( W \) and \( \alpha \), the solution for \( \alpha \) can only be determined by numerically solving (45). The computational procedure for \( \alpha \) is done by setting up a grid that ranges from 0.00001 to 0.999 with increments of 0.001. The optimal value of \( \alpha \) is found by evaluating the first-order condition at different values of \( \alpha \) to find the point at which (45) holds with equality.
utilities. Again, since it is not possible to do so analytically, we resort to numerical solutions to determine the agent’s decision which are presented next.

**Numerical Experiments**

We first start by considering the distribution of wealth and the proportion of unreported income ($\alpha$) in the two models. Figure 11 plots the proportion of unreported income for individuals in the AS economy and ‘evade or not’ economy for a value of $\theta=0.15$ and a Gini-coefficient of 0.3439. The blue line represents the AS model and the green dots represent the ‘evade or not’ model. An interesting outcome that has emerged is that the proportion of unreported income ($\alpha$) now varies with wealth. This is in stark contrast to the models without a cost-of-evasion function where the proportion of reported income $X$ was a constant with respect to wealth. This result seems more realistic as we would expect richer agents to evade a higher proportion of their income.\(^{19}\)

The distribution of evaders has also changed. It is now the richer agents in the economy who evade taxes while the poorest agents do not evade and report their full income. We can also see that the extent of evasion increases with wealth in both models. First consider evasion in the AS model. Here the proportion of unreported income by the wealthiest agents is around 0.055, whereas the proportion of unreported income by the lower income agents is around 0.015. This result is substantially lower than the standard AS formulation and the proportion of income evaded is much more realistic in this instance. For example, according to Bloomquist (2003), the IRS estimated that taxpayers who filed returns reported about 99% of all wage income and those with non-farm proprietor income reported about 67.7%.

In the ‘evade or not’ economy, however, we can see that a large number of agents choose not to evade taxes, reducing the extent of evasion in another sense, and thereby taking the model a step further to what is observed empirically. In a sample of 501 agents only 110 of the richest agents choose to evade. Once again, this result is more consistent with the empirical evidence. As mentioned earlier, according to Christian (1994), 60% of U.S taxpayers do not understate their income. In our ‘evade or not’ model, about 78% of agents reported their true income and do not evade from the payment of taxes.

\(^{19}\) This is the reason for which a large number of tax evasion studies assume DRRA preference (see for example, Jung et al. 1994). For our purposes, as emphasised earlier, we wish to preserve the CRRA assumption discussed in Section 1.
We then turn to the effect of inequality on the number of evaders in the ‘evade or not’ model (recall that in the AS model all agents in the economy evade). We can see from Table 3 that different levels of inequality give rise to different outcomes for $\theta=0.15$ and $\theta=0.2$.

The extent of evasion in the economy does indeed change when inequality varies. However, the effect of inequality seems to be non-monotonic with respect to the number of evaders in the economy. For example, the number of evaders increases from 22 to 110 when inequality rises from 0.2735 to 0.3439 but falls to 20 when the Gini-coefficient is raised further to 0.4073. Interestingly, the introduction of a cost-of-evasion function seems to have increased the number of evaders in the economy in comparison to the model without a cost-of-evasion function. This could be due to the interactions from the non-linear relationship between the cost of evasion, the proportion of unreported income, and the tax rates.

\footnote{Note that with the introduction of a cost-of-evasion function, the range of parameters is longer restricted to the conditions for an interior solution given in the previous section. Our selection for the range of $\theta$ in these experiments relate to realistic values of tax rates that is observed around the world.}
Table 3: Number of Evaders for Different Levels of Inequality with Cost of Evasion

<table>
<thead>
<tr>
<th>θ</th>
<th>Gini=0.2735</th>
<th>Gini=0.3439</th>
<th>Gini=0.3807</th>
<th>Gini=0.4073</th>
<th>Gini=0.5895</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
</tr>
<tr>
<td></td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
</tr>
<tr>
<td>0.15</td>
<td>22</td>
<td>110</td>
<td>0</td>
<td>20</td>
<td>9</td>
</tr>
<tr>
<td>0.20</td>
<td>496</td>
<td>495</td>
<td>501</td>
<td>330</td>
<td>501</td>
</tr>
<tr>
<td>0.25</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.30</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.35</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.40</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.45</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.50</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.55</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.60</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.65</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.70</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ</th>
<th>Gini=0.5975</th>
<th>Gini=0.6736</th>
<th>Gini=0.7143</th>
<th>Gini=0.8346</th>
<th>Gini=0.8388</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
<td>No. of Evaders</td>
</tr>
<tr>
<td></td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
<td>Evade or Not Model</td>
</tr>
<tr>
<td>0.15</td>
<td>13</td>
<td>37</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.20</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.25</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.30</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.35</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.40</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.45</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.50</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.55</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.60</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
<tr>
<td>0.65</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
<td>501</td>
</tr>
</tbody>
</table>
Next, we analyse the indirect utility functions of the agents over their preferences of $\theta$ in the AS economy. The results are presented in Figures 12 and 13. Immediately, we can see that the utility functions of the agents are single-peaked once the cost-of-evasion parameter is incorporated in the model. This is in contrast to the previous model (the model without a cost of evasion) where the indirect utility functions of the agents were non-single peaked. In the ‘evade or not’ model, we can see that the preferences of the agents over the tax rate are also single peaked. As a result, the voting outcomes and distribution of votes may be different to the model without a cost associated with tax evasion (which will be explored shortly).

In addition, the indirect utility functions show that agents on the low and middle end of the income distribution prefer relatively high tax rates, whereas the richer agents prefer relatively low tax rates. This is due to the effect of redistributive transfers discussed earlier. In addition, in the ‘evade or not’ model, Agent 351, seems to prefer tax rates that are in the middle of the spectrum as opposed to the ends. A tax rate of $\theta=0.4$ gives Agent 351 the highest utility from the considered range. This suggests that the pattern of votes, in terms of the percentage of agents in favour of any particular tax rate, may be different relative to the previous model.
Figure 12: Agents’ preferences over $\theta$ in the AS economy.

Figure 13: Agents’ preferences over $\theta$ in the ‘Evade or Not’ economy.
Our intuition is confirmed by the results presented in Figures 14 and 15, which present the results of voting over different values of \( \theta \). As expected, the inclusion of a cost of evasion has not altered the winning outcome of \( \theta \); redistribution is favoured in both models in the presence of inequality. Furthermore we get stronger results in favour of such outcomes given the single peakedness of the preferences. In this case, we may simply look at the median agent’s (i.e. agent 251’s) preference to determine the voting outcome. The majority of votes in both economies are in favour of the highest tax rate of \( \theta = 0.65 \). The second preferred tax rate in both economies is \( \theta = 0.15 \). This is in contrast to the ‘evade or not’ model without a cost-of-evasion function where the second preferred tax rate was \( \theta = 0.2 \). The distribution of votes in both the AS model and the ‘evade or not’ alternative is also almost identical. Figures 14 and 15 below present the results.

Figure 14: Percentage of votes for various values of \( \theta \) in the AS economy.
Finally, in Table 4 below, we look at the voting outcomes on $\theta$ with various levels of inequality with the introduction of a cost-of-evasion function. We can see that in both the AS and ‘evade or not’ model, increasing the level of inequality does not alter the voting outcome of the tax rate. The voting outcomes of the models are identical, with agents in the economy voting for the highest tax rate available to them of $\theta=0.65$. This is in contrast to the model without cost of evasion presented in the previous section where for an inequality level of Gini=0.2735, the vote in the ‘evade or not’ model was $\theta=0.20$. The percentage of votes in both the AS and ‘evade or not’ model are also identical, and in this instance we get a majority outcome where the winning value of $\theta$ is always greater than fifty percent. This is in contrast to the extensions without a cost-of-evasion function where the outcomes were the result of a plurality vote.
Table 4: Vote on $\theta$ for Different Levels of Inequality with Cost of Evasion

<table>
<thead>
<tr>
<th>Gini</th>
<th>Vote on $\theta$</th>
<th>% in Favour</th>
<th>Vote on $\theta$</th>
<th>% in Favour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Evade or Not)</td>
<td>(Evade or Not)</td>
<td>(AS model)</td>
<td>(AS model)</td>
</tr>
<tr>
<td>0.2735</td>
<td>0.6500</td>
<td>58.4830</td>
<td>0.6500</td>
<td>58.4830</td>
</tr>
<tr>
<td>0.3439</td>
<td>0.6500</td>
<td>66.6667</td>
<td>0.6500</td>
<td>66.6667</td>
</tr>
<tr>
<td>0.3807</td>
<td>0.6500</td>
<td>65.0699</td>
<td>0.6500</td>
<td>65.0699</td>
</tr>
<tr>
<td>0.4073</td>
<td>0.6500</td>
<td>68.0639</td>
<td>0.6500</td>
<td>68.0639</td>
</tr>
<tr>
<td>0.5895</td>
<td>0.6500</td>
<td>72.2555</td>
<td>0.6500</td>
<td>72.2555</td>
</tr>
<tr>
<td>0.5975</td>
<td>0.6500</td>
<td>72.2555</td>
<td>0.6500</td>
<td>72.2555</td>
</tr>
<tr>
<td>0.6736</td>
<td>0.6500</td>
<td>74.4511</td>
<td>0.6500</td>
<td>74.4511</td>
</tr>
<tr>
<td>0.7143</td>
<td>0.6500</td>
<td>79.0419</td>
<td>0.6500</td>
<td>79.0419</td>
</tr>
<tr>
<td>0.8346</td>
<td>0.6500</td>
<td>86.8263</td>
<td>0.6500</td>
<td>86.8263</td>
</tr>
<tr>
<td>0.8388</td>
<td>0.6500</td>
<td>86.0279</td>
<td>0.6500</td>
<td>86.0279</td>
</tr>
</tbody>
</table>

7. Concluding Remarks

The key objective of this paper was to provide the necessary first steps in the modelling of tax evasion within a macroeconomic framework. This involves the construction of a model with heterogeneous agents and redistributive transfers, so that the implications of tax evasion may be considered. The political economy angle is then modelled in a simple way by allowing the agents to vote on their desired tax structure. More importantly, the framework we construct incorporates the idea that agents typically face various trade-offs that can only be realistically modelled within a macroeconomic framework.

The results of our analysis lead to some interesting insights. The introduction of the ‘Evade or Not’ feature of the model is a key contribution to the literature because it reduces the extent of evasion even in the context of a very simple macroeconomic model of tax evasion. We find that the extent of evasion in the ‘evade or not’ alternative is much lower and more consistent with the empirical evidence. Another realistic outcome that emerges is that the extent of evasion is increasing in wealth. Typically, tax evasion studies have to resort to DRRA preferences to achieve levels of evasion that are increasing in wealth, a feature that has some empirical support in the literature. In the context of the model of this paper, this is achieved while still maintaining CRRA preferences. This is important in the sense that macroeconomic models require preferences to be restricted to the CRRA class if they are to be consistent with some stylised facts pertaining to business cycles and economic growth.
There is also a reduction of the extent of evasion in another sense: the percentage of evaders in the model economy is also reduced to numbers that are more consistent with the empirical estimates in the literature.

In addition, for a range of values of the tax rate, the ‘evade or not’ model always produces a lower amount of evasion in comparison to the AS model. Furthermore, an interesting outcome emerges in relation to the mix of evaders in the distribution. For low levels of the tax rate evasion is concentrated at the bottom end of the income distribution and this tendency is exacerbated when inequality rises. When we introduce a cost-of-evasion function, we see that the identity of evaders in the distribution have now switched. It is now the richer agents rather that the poor agents who evade from the payment of taxes. The results also show, that in this instance, the effect of inequality seems to be non-monotonic with respect to the number of evaders in the economy.

The political economy outcomes of the models are also of interest. We find that in the vast majority of cases, redistribution is favoured in both the AS model and the ‘evade or not’ model in the presence of inequality. One notable exception is for one special case of the ‘evade or not’ construct without a cost-of-evasion function for a very low level on inequality. In this instance, we find that the agents prefer ‘efficiency over equity’ and vote on a low level of progressivity. Finally, we find that the level of inequality does not seem to matter in relation to the tax structure.
References


Appendix

Appendix A: Comparison of Indirect Utility IUFAS and IUFNE.

Comparing the indirect utilities of the AS model (labeled IUFAS) and the model without evasion (labeled IUFNE), and assuming log utility, gives the following:

\[ IUFAS \geq IUFNE \iff \]

\[
(1 - p)ln(1 - \delta\theta) + pln(1 - \delta\theta - \pi(1 - \delta)) \\
\geq pln(1 - \theta) + (1 - p)ln(1 - \theta) \\
\Rightarrow (1 - \delta\theta)^{1-p}(1 - \delta\theta - \pi(1 - \delta))^p \geq (1 - \theta)^p (1 - \theta)^{1-p} \\
\Rightarrow (1 - \delta\theta)^{1-p}(1 - \pi + \delta(\pi - \theta))^p \geq (1 - \theta)^{1-p}(1 - \theta)^p
\]

(A1)

Recall that \( \delta = \left(\frac{\pi p - \theta + \pi \theta (1-p)}{\pi \theta - \theta^2}\right) \).

For the indirect utility in the AS model to always be greater than the indirect utility of the not-evade alternative, we would also require the second term on the LHS to greater than the second term on the RHS.

\[ IUFAS \geq IUFNE \iff \]

\[
(1 - \delta\theta)^{1-p}(1 - \delta\theta - \pi(1 - \delta))^p \geq (1 - \theta)^p (1 - \theta)^{1-p} \\
\Rightarrow \theta(1 - \pi) + \pi p - \theta + \pi \theta(1 - p) \geq (1 - \theta)\theta \\
\Rightarrow \pi p \geq \theta
\]

(A2)

If the conditions for an interior solution are satisfied, however, the second term on the LHS is less than the second term on the RHS, making it difficult to compare the expressions of both side of the inequality.
Appendix B: Derivation of Conditions for an Interior Solution.

For an interior solution:

\[
0 < \frac{\pi p - \theta + \pi \theta (1 - p)}{\pi \theta - \theta^2} < 1
\]

\[
\Rightarrow 0 < \pi p - \theta + \pi \theta (1 - p) < \pi \theta - \theta^2
\]

(A3)

This implies that for \( X < W \) (upper bound condition):

\[
\pi \theta - \theta^2 > \pi p - \theta + \pi \theta - \pi \theta p
\]

\[
\Rightarrow \pi p < \theta
\]

(A4)

and for \( X > 0 \) (lower bound condition):

\[
0 < \pi p - \theta + \pi \theta - \pi \theta p
\]

\[
\Rightarrow \frac{\theta (1 - \pi)}{(1 - \theta) \pi} < p
\]

(A5)

which gives:

\[
\frac{\theta (1 - \pi)}{(1 - \theta) \pi} < p < \frac{\theta}{\pi}
\]

(A6)
Appendix C: Comparison of Indirect Utility IUFAS and IUFNE Two-Period Model.

Comparing the indirect utilities of the AS two-period model (IUFAS) and the ‘evade or not’ two-period model (IUFNE) gives the following:

\[ \text{IUFAS} \geq \text{IUFNE} \text{ iff } \]

\[
\ln(1 + r) + 2p \ln \left( \frac{1 - \theta \frac{X'}{W} (1 - \pi)W}{2} \right) + 2(1 - p) \ln \left( \frac{1 - \theta \frac{X'}{W} W}{2} \right) \\
\geq \ln(1 + r) + \ln(1 - \theta)
\]

Rearranging terms we get:

\[
2p \ln \left( \frac{1 - \theta \frac{X'}{W} (1 - \pi)W}{2} \right) + 2(1 - p) \ln \left( \frac{1 - \theta \frac{X'}{W} W}{2} \right) \\
\geq \frac{\ln (1 - \theta)W}{2}
\]

\[
2\ln \left( \frac{1 - \theta \frac{X'}{W}}{2} \right) + 2p \ln (1 - \pi) \geq 2 \ln \frac{(1 - \theta)}{2} \quad (A7)
\]

Comparing the utilities of the two models, it is again not possible to prove the proposition that the utility with evasion (IUFAS) is higher than the certainty scenario (IUFNE) given that the conditions for an interior solution are satisfied.