Contacts, Market Institutions, and Development

Paul Frijters, Dirk J Bezemer, Uwe Dulleck

Working/Discussion Paper # 205a
March 2005

Abstract:

We propose an endogenous growth model that incorporates the importance of business contacts and informal contacts. In our model, sold output increases with the stock of business contacts. The modelling of contact creation is based on matching theory. The cost of creating contacts decreases with more Community level Social Capital and Market Institutions, which we understand as networks of informal contacts.

Technological growth is driven by the replacement of contacts within the economy. Political interference and centralization can provide disincentives to break old contacts and hence affect innovation. Simulations suggests that our model is in line with empirical observations.

Classification JEL codes: O11, O41, P51

Keywords:  Endogenous Growth, Relational Capital, Development, Economic Systems, Social Capital
Contacts, Market Institutions, and Development

Paul Frijters
Australian National University

Dirk J. Bezemer
University of Groningen

Uwe Dulleck*
University of Vienna

March 2005

Abstract

We propose an endogenous growth model that incorporates the importance of business contacts and informal contacts. In our model, \textit{sold} output increases with the stock of business contacts. The modelling of contact creation is based on matching theory. The cost of creating contacts decreases with more Community level Social Capital and Market Institutions, which we understand as networks of informal contacts.

Technological growth is driven by the replacement of contacts within the economy. Political interference and centralization can provide disincentives to break old contacts and hence affect innovation. Simulations suggest that our model is in line with empirical observations.

\textbf{Keywords:} Endogenous Growth, Relational Capital, Development, Economic Systems, Social Capital

\textbf{JEL:} O11, O41, P51

*Corresponding author: Uwe Dulleck, Hohenstaufengasse 9, 1010 Vienna, Austria; uwe.dulleck@univie.ac.at. We gratefully acknowledge helpful comments by Michael Ellman, Ruud Knaack, Robert Scharrenborg and Gerhard Sorger.
1 Introduction

In this paper we propose an endogenous growth model that details the role of business contacts in development.\footnote{While this paper discusses development broadly defined, in a companion paper we also apply the main ideas specifically to transition economies (Bezemer et al, 2003).} We draw attention to the fact that search frictions make existing contacts between market parties valuable and give an indirect productive role to all the networks and institutions that ease these frictions. Business contacts - representing knowledge about the existence, reliability and trustworthiness of potential trading partners (suppliers and clients) - are a productive input and require labour time to find. We name these contacts Relational Capital. Some of these potential trade links become obsolete in the process of technological progress. The termination of contacts by one transaction partner has a negative external effect on other partners.

The externality of technological upgrading is inspired by the ‘disorganisation’ models of Blanchard and Kremer (1997) and Roland and Verdier (1999) who relate output falls of whole economies to the negative externality of break-ups between trading partners. In these models though, firms have no explicit dynamic stock of productive relations and there is no labour cost of making contacts.

Our view of the process of innovations differs subtly from existing arguments in the endogenous growth theory. As in Romer (1986, 1994) we assume that innovation occurs on the level of the individual firm. We add the idea that innovation requires the destruction of some of the business contacts of a firm because new products require new suppliers and new clients. This is a form of creative destruction, where established business contacts are terminated by one party in the process of innovation. Our notion of creative destruction does not focus on products made obsolete but on contacts made obsolete. Our view agrees with the observation of Aghion and Howitt (1998, pg. 1) that “economic growth involves a two-way interaction between technology and economic life: technological progress transforms the very economic system that creates it”. 
Economies in our model can differ in several respects. First, the labor cost of creating Relational Capital may be different. That is, for reasons of existing networks or institutions firms in an economy may face lower labor cost to establish new contacts. Second, economies may differ in the embedded opposition against the replacement of contacts - which is in our eyes necessary for technological advancement. Political interference, including corruption and lobbying, as well as centralization reduce the efficiency of firms to seek new contacts to replace old ones. Opposition towards the replacement of contacts increases the relative labor cost of contact replacement compared to the addition of new contacts.

Our Model bears some parallels with the literature on social capital. In this literature there exists a dichotomy between an individual level and a community level concept of social capital. The concept of individual level social capital concurs with Arrow’s (1999) demand that capital should be something that is individually accumulated. Glaeser et al. (2002) provide a nice illustration of the ”size of a rolodex” as a measure of the social capital an economic agent holds. Putnam (2000) and Burt (2000) see the size of an agents network as a measure of his or her social capital. In this respect, Relational capital can be interpreted as individual level Social capital. Community level social capital does not equate with assets of individuals but exists at a community level. Bowles and Gintis (2002), Knack and Keefer (1997), Nooteboom (2002), Robison et al. (2002), Sobel (2002) are proponents of this view. They all see that trust, ”sympathy among agents” etc. to facilitate the creation of productive contacts. through this channel, they argue it enhances the economic growth of an economy. We argue below that high levels of community level social capital, i.e. trust, decrease the labor cost of finding new contacts, which can be added to the stock of contacts or used to replace ”old” contacts. Market Institutions (North, 1990) have a similar effect.

We set out our model in the next section. In section 3 we provide some further evidence from the literature that support our model. Section 4 presents
a series of simulations to get an understanding of role of RC and Community Social Capital for different development paths and political environments. We distinguish between exogenous ‘big bang’ transitions, exogenous transitions of slow but inevitable change, and endogenous developments. Section 5 concludes and raises issues for further research. The Appendix provides micro-foundations for most parts of our model and shows how we calculated the simulations.

2 A Model of Relational Capital and Growth

We define a continuum of representative profit-maximizing firms. Consumption is not explicitly considered, but firms can be viewed as owned by households who provide a fixed endowment of labor to the economy. Households consume all of their income except a constant share $s$ as specified below. Firms produce a homogeneous good with unit price. Technology is described by a production function with three inputs: labor, physical capital and contacts. Thus, Relational Capital ($RC_t$) is a capital stock, and can be thought of as the number of business contacts. It is an input in sold output $y_t$.

The difference to the standard definition of output is that market frictions necessitate business contacts. Having RC as an input is our way of modelling the search costs of finding partnerships needed for buying inputs and selling output. We define sold output by

$$y_t = y(A, L_t - L_t^{rc}, RC_t, K_t)$$

where $y_t$ is sold production at time $t$; $L_t$ is the labor force; $L_t - L_t^{rc}$ is net labor input into physical production ($L_t^{rc}$ is labor devoted to the creation of $RC_t$); $A_t$ is the technology parameter; $K_t$ is physical capital. $y(.)$ is a constant-returns-to-scale function with all the usual Inada-properties: any input faces decreasing positive marginal returns and is technically complementary to any other input.

The economy has a continuum of such firms with a measure of 1. This
allows us to use $\bar{y}_t$, $\bar{L}_t$, $\bar{K}_t$, and $\bar{RC}_t$ as the total amount of output, labor and capital stocks in the whole economy. As in standard macroeconomic growth models we assume the following functional form for our analysis

$$y_t = y(A_t f(L_t - L_t^{rc}, RC_t), K_t)$$

(2)

where $A_t f(L_t - L_t^{rc}, RC_t)$ is a single composite input: technology $A_t$ is the productivity of the combination of labor and contacts, similar to a labor augmented (or Harrod-neutral) technology in the standard textbook model. Assumptions on $f(.)$ are implicitly given by the assumptions on $y(.)$.

Firms select levels of $L_t$ and $K_t$ and invest in the stock of $RC_t$ by allocating labor $L_t^{rc}$. We distinguish between $D_t^{rc}$, the amount of contacts replaced, and $N_t^{rc}$, the amount of contacts added. Replacing contacts implies destroying an old contact and creating a new one, as illustrated in figure 1 (page 8) below.

Firms selecting positive levels of $D_t^{rc}$ and $N_t^{rc}$ meet on a market for contacts. Firms, and therefore business contacts, are taken to be heterogeneous, leading to search frictions in the matching process. As in most of the search literature (e.g. Pissarides, 2000; Petrongolo and Pissarides, 2001; and Frijters and Van der Klaauw 2005), we do not explicitly model this heterogeneity. We capture its effect by positing contact search costs in terms of labor time to replace or add business contacts:

$$\lambda_t L_t^{rc} = \varphi_t D_t^{rc} + N_t^{rc} \text{ with } \lambda_t > 0, \varphi_t \geq 1$$

(3)

where $\lambda_t$ denotes the conversion rate of labor $L_t^{rc}$ into relations. In terms of search theory, $\lambda_t$ can be interpreted as the arrival rate of contacts. We capture the relation between business contacts and social or market networks by positing that $\lambda_t$ depends positively on both Community Social Capital and

---

2 This differs from social network models such as Jackson and Wolinsky (1996), Vega-Redondo (2003) or the growth model by Routledge and von Amsberg (2003). In those models the stability and/or trustworthiness of specific links between agents is analyzed. We abstract from identities of partners by assuming these problems are captured implicitly by a matching function.
Market Institutions. In the next section, we elaborate on this and provide references to the literature.

Since destroying an old contact constitutes a negative externality (the loss of the value of a contact) to the old business partner, these have an incentive to pre-empt by making contact destruction costly. \((\varphi_t - 1)\) is the cost a firm incurs when breaking a contact with another firm, over and above the costs of just finding a new contact. We assume that raising the cost of breaking contacts is possible via the political process. If there is some degree of political interference in firms’ matching choices, \(\varphi_t \geq 1\) denotes the degree to which the political process frustrates the replacement of contacts. In completely decentralized economies, firms have no power to raise the cost of breaking contact and \(\varphi_t = 1\): replacing and adding contacts are equally costly to the firm doing it. Political interference in matching choices amounts to some degree of centralization of markets. The more an economy is centrally controlled, in this sense, the higher \(\varphi_t\). We discuss this assumption in the next section in more detail.

Contact replacement is inextricably linked to technological progress. Whenever a firm increases its efficiency by initiating a new production method, producing new products, or changing its internal organization, it will typically make new demands on its input suppliers or output buyers. Switching transaction partners will be optimal under new production or sale conditions since old ‘transaction partners’ were selected so as to match old production and sale processes. Firms tend to replace contacts as they improve their technology. As in Schumpeter (1934) and Routledge and von Amsberg (2003), the destruction of old contacts is an inevitable by-product of the creation of new production and sale methods. We therefore term the replacement of RC

---

3 Routledge and von Amsberg (2003) provide a game theoretic model of SC based on the idea of cooperation in a repeated Prisoner Dilemma game. To model growth they assume too that new trading partners are necessary for technological advancement. In their model, faster technological development implies shorter times of interaction and hence a destruction of Community Social Capital in the form of trust. We argue that only RC diminishes through an externality of replacing contacts. In our model Community
creative destruction. We explicitly model technological progress as depending on the extent of contact replacement $D_t^{rc}$:

$$A_t = A_{t-1} + (A^*_t - A_{t-1})g(D_{t-1}^{rc})L_t$$  \hspace{1cm} (4)

where $A^*_t$ denotes the production frontier at time $t$. The function $g(.) \geq 0$ denotes technological ‘catch-up’ resulting from the replacement of $RC$ per unit of $L_t$. The lag between $D_t^{rc}$ and $A_t$ reflects the technology take-up time. We assume that there are decreasing returns in technological investment:

$$\frac{\partial g(.)}{\partial D_t^{rc}} > 0, \quad g(0) = 0, \quad \text{and} \quad \frac{\partial^2 g(.)}{\partial D_t^{rc}^2} < 0.$$  \hspace{1cm} Appendix 1.3 provides micro-arguments for this equation.

Because of the externality connected to replacing contacts, the level of $RC$ does not only depend on own investment decisions, but also (negatively) on others’ contact replacement decisions. We capture this by

$$RC_t = RC_{t-1}e^{-\beta\frac{D_t^{rc}}{N_{t-1}}} + N_t^{rc}$$  \hspace{1cm} (5)

where the term $e^{-\beta\frac{D_t^{rc}}{N_{t-1}}}$ equals the probability of an old contact being destroyed by the creative destruction decisions of other firms. This probability is derived endogenously given a stochastic process on the micro level which we develop in detail in Appendix 1.1. The parameter $\beta$ equals the net number of contacts that get destroyed when one firm replaces an old contact, destroying his previous partner firm’s contact. When that firm is part of a large value chain of interdependent firms, $\beta$ is large.

Figure 1 illustrates the difference between $N_t^{rc}$ and $D_t^{rc}$. For simplicity, we take $\beta = 1$. This reflects the simplifying assumption that production is pairwise, i.e. that value chains have a length of two firms. There are four firms in total. Initially, there are productive contacts between firms 1 and 2, and between firms 3 and 4. The top example shows what happens with creative destruction: firms 1 and 3 both replace one contact and form a new contact through search and matching. Both firms improve their technology.

---

Social Capital can help to reduce the cost of the externality much in line with empirical evidence (see for example Miguel’s (2003) comment on Routledge and von Amsberg).
Figure 1: Creative destruction (replacing an old contacts) and network extension

$A_t$ by doing so. Both abandon the contact they previously had with other entities. The net effect of this creative destruction is a loss of one contact. As noted, we can extend this example to situations where the net number of contacts that are destroyed is larger. If some of these entities are part of a chain of contacts, the whole chain may become worthless when a single entity in the chain pulls out. The bottom example shows what happens with making extra contacts: without changing production processes, both entities 1 and 3 increase their number of contacts. The new contact between these entities does not force either of them to abandon their previous contacts. The net effect is an increase in the number of contacts by one.

To close our model, we make some standard assumptions about the movement of total labor units, the technological frontier and physical capital formation:
\[ L_t = L \]
\[ K_t = (1 - \delta)K_{t-1} + s y_{t-1} \]
\[ A_t^* = (1 + \alpha)A_{t-1}^* \]

We take labor to be constant and capital to follow the Swan-Solow assumptions of fixed depreciation, constant savings rate and exogenous technological frontier progress. This specification reflects assumptions on the economy of exogenous savings, no outside investment and a given technological frontier.

We make the standard assumption that firms maximize the discounted stream of output equal to \( \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t y_t - wL - rK_t \). This is independent of the economic system, which is here reflected in the centralization parameter \( \varphi_t \). This implies that we assume that economic systems do not affect optimization behavior, but they do affect the constraints firms face.

3 Support from the literature: Social capital, Institutions and Growth

We suggest that the costs of making new contacts and replacing old ones are determined by three factors: Community Social Capital, the quality of market institutions, and the political process. In this section we provide some evidence backing our assumptions.

We model Community Social Capital as the size of the informal network within a community. Contacts facilitate information exchange; as Malecki (2000) writes, ‘through the economic and social relationships in the network, diverse information becomes less expensive to obtain’. Nooteboom (2002) argues that degree of trustworthiness of informal networks, such as family, ethnic, religious, and civil ties, is larger because participants have non-profits reasons to interact. Therefore these networks provide means of obtaining at lower cost information about potential “trustworthy” trading partners. That
informal networks facilitate creation of productive and innovative contacts is illustrated empirically by Murphy (2002), who reports that social networks of business people in Tanzania support innovation in manufacturing firms. He documents that the main reason is an improved quality of information exchange. The positive link between informal trust and growth is well established in the empirical literature (Knack and Keefer, 1997).

In the simulation section below and the appendix 1.2 we model Communal Social Capital to carry some (positive) network externalities. An argument inspired by Diamond (1982) and Howitt and McAfee (1992). The social capital literature provides a similar - though often not formalized - argument about informal networks and trust. Sobel (2002) is one example. He argues that growth of the overall network increases the contact rate, which spurs further growth of the network. Such a self-enforcing mechanisms can accelerate growth but it can also cause a downward spiral caused by an exogenous drop in network size.

In our model Market Institutions⁴ are a close substitute to Community Social Capital. Market Institutions also determine the costs of making contacts because by reducing the cost to acquire information about potential trading partners. Demirguc-Kunt and Maksimovic (2002) provide empirical support with respect to financial institutions in developing countries. With financial transactions, information about the creditworthyness of trading partners matters. This can be provided on a small scale via social networks, on a larger scale only formal institutions are able to provide such information.

We now turn to the link between the political system and contact replacement. There are three reasons why political-economic systems may deviate from the assumption that replacing a contact is as (labor) costly as adding a new contact. The first is lobbying and corruption. Politicians and bureaucrats may be paid by potential losers of creative destruction to stop firms planning to replace contacts. Similarly, in a highly centralized system,

⁴We use the word institution not merely for the rules that these formal organizations enforce (as in North, 1990), but also for the organizations themselves.
firms negatively affected by creative destruction can lobby the political center not to allow creative destruction in other firms, thus making the state a stakeholder in their interests. Such lobbying has indeed been observed in developing and transition economies (Rama, 1993; Braguinsky and Yavlinsky, 2000; Gros and Steinherr, 1995) and was prevalent in socialist systems (Nove 1987).

The second reason is nepotism, when politicians and bureaucrats hold a stake in firms, other firms will consider twice to replace a contact: one does not easily break up with the dictator’s firm. A similar argument is made by Agesa (2000).

A third way in which the political-economic system may raise the costs of breaking contacts is economic planning. The literature on central planning argues that the span of control of the center is typically not sufficient to gather and absorb all the information necessary to make optimal enterprise-level decisions, among them decisions on the breaking and making of contacts. Especially the recognition of new technological opportunities is a matter of local information. For socialist economies, this argument is well-known (e.g. Aslund 2002), and socialist-style central planning constitutes the extreme case. But also milder cases of economic planning imply that contact selection by politicians prevails over contact selection by firms. These are likely to be sub-optimal contact choices.

In some of the simulation section below, we assume a feedback from higher $RC$ to $\varphi_t$. That contacts can be used as channels of information and manipulation, and are therefore a means to influence politics is argued in Guy (2000). Firms can employ their contacts to fight for better market institutions and less corrupt and less ‘influential’ politicians.

\footnote{\textsuperscript{5}likely it is that those at the firm level can reap the benefits of improved technology and replaced contacts.}

\footnote{\textsuperscript{6}For a more developed model on this specific issue, see Dulleck and Frijters (2003), who stress the importance of rents from a resource sector (e.g. oil or minerals) to the behavior of politicians.}
4 Scenarios of Economic Development

Intuitively, the growth properties of any steady state appear trivial: as long as $\phi < \infty$ and $RC_t$ cannot grow to infinity, then all steady states under all specifications will have a growth rate equal to the rate of exogenous technological change. Since the central problem in development is the uneven distribution of growth in the medium run, we are more interested in transition paths under specific parameter assumptions. We thus only briefly discuss the maximisation problem, the steady state, and the computation method for calculating transition paths. Most of the details are relegated to Appendix 2.

4.1 The Maximisation Problem and Steady States

We try to remain as close to mainstream models as we can in order to be able to use the findings of other studies as sources of reasonable parameters. Our functional form specification for output and technology growth is:

$$y_t = [A_t(L - L'^t)^{\lambda_0} R C_t^{1-\gamma_0})]^\gamma K_t^{1-\gamma}$$

$$A_t = A_{t-1} + g_t (1 - e^{-g_0 D_t-1}) (A_t^* - A_{t-1})$$

which presumes a standard Cobb-Douglas production function and a simple catch-up process for technological progress.

We here discuss the simplest version of the model, i.e. with a fixed $\phi$ and $\lambda$. Define $\mu_t = \frac{A_t}{A_{t-1}}$. In case of interior solutions, the maximization problem of the representative agent is
\[
\max_{L_t, K_t, D_t^{rc}, N_t^{rc}} U_s = \sum_{t=s}^{\infty} e^{-\rho(t-s)} \left[ \mu_t A_0^s (1 + a)^{t-1} (L_t - L_t^{rc})^{\gamma_0} RC_t^{1-\gamma} \right] \gamma K_t^{1-\gamma}
\]

\[
\lambda L_t^{rc} = \varphi D_t^{rc} + N_t^{rc}
\]

\[
\mu_t = \frac{\mu_{t-1}}{1 + a} + g_1 (1 - e^{-\rho_0 D_{t-1}}) (1 - \frac{\mu_{t-1}}{1 + a})
\]

\[
RC_t = RC_{t-1} e^{-\beta \frac{\partial y_{t+1}}{\partial t-1}} + N_t^{rc}
\]

\[
K_t = (1 - \delta) K_{t-1} + s y_{t-1}
\]

The two state variables in this model are then \(\mu_{t+1}\) and \(RC_t\). Solving out the non-state variables begets

\[
\max_{\mu_{t+1}, RC_t} U_s = \sum_{t=s}^{\infty} e^{-\rho(t-s)} \left[ \mu_t A_0^s (1 + a)^{t-1} (L_p)^{\gamma_0} RC_t^{1-\gamma} \right] \gamma [(1 - \delta) K_{t-1} + s y_{t-1}]^{1-\gamma}
\]

where \(L_p = (L_t - \frac{\varphi - \frac{1}{\rho_0} \ln(1 - \frac{\mu_{t+1} - \frac{\mu_0}{1+a}}{g_1(1-\frac{\mu_{t+1}}{1+a})}) + RC_t - RC_{t-1} e^{-\beta \frac{\partial y_{t+1}}{\partial t-1}}}{\lambda})
\]

\[
D_t^{rc} = -\frac{1}{\rho_0} \ln(1 - \frac{\mu_{t+1} - \frac{\mu_0}{1+a}}{g_1(1-\frac{\mu_{t+1}}{1+a})}) \text{ and } N_t^{rc} = RC_t - RC_{t-1} e^{-\beta \frac{\partial y_{t+1}}{\partial t-1}}
\]

where \(y_{t-1}\) of course depends on \(\mu_t, \mu_{t-1}, RC_t, \text{ and } K_{t-1}\). The first-order conditions yielding Euler equations are then

\[
\frac{\partial U_s}{\partial \mu_{s+1}} = \frac{\partial y_s}{\partial \mu_{s+1}} + e^{-\rho} \frac{\partial U_{s+1}}{\partial \mu_{s+1}} + e^{-2\rho} \frac{\partial U_{s+2}}{\partial \mu_{s+1}} = 0
\]

\[
\frac{\partial U_s}{\partial RC_s} = \frac{\partial y_s}{\partial RC_s} + e^{-\rho} \frac{\partial U_{s+1}}{\partial RC_s} + e^{-2\rho} \frac{\partial U_{s+2}}{\partial RC_s} = 0
\]

These Euler equations can be used to find candidates for stationary steady states \(\{\mu_{s+1}, RC_s\}\) that satisfy these necessary conditions. In all the situations we examined there turned out to be only one candidate. These stationary steady states were then numerically checked for stability, and used
to find transition paths. Our general approach for this was to presume the steady state was reached some point in the far future (e.g. after 200 periods), and solve the transition path backwards.

Finding transition paths is complicated by the presence of non-negativity constraints on $N^r_t$ and $D_t$. These constraints in practise turn out to be binding in some periods. One would in principle have to check $2^{2\times200}$ combinations of constraints for a transition path of 200 years with 2 possibly binding constraints each period. The clear infeasibility of doing this forces us to use approximating heuristics to find transition paths. Our method, which in effect relies on using non-linear optimisation routines to directly find the Rational Expectations optimum of $U_s$, is described in depth in Appendix 2.

4.2 Parameter choices

We initially take: $\gamma_0 = 0.65$, $\gamma = 0.7$, $g_0 = 0.25$, $g_1 = 0.8$, $\lambda(.) = \lambda_0 = 0.1$, $\rho = 0.06$, $s = 0.3$, $\beta = 1$, and $a = 0.02$. We later discuss alternative scenarios and change $\lambda_t$ accordingly.

As to the initial condition, we presume in all scenarios that the economy starts with $\varphi_t = \varphi = 1000$. The gap with the technological frontier at the start of each development trajectory is presumed equal to 100 years of steady state technological development. At 2 percent technological growth per year, this works out at a technological ratio of about 1:7, which appears a reasonable guesstimate. We note that the productivity per unit labor has a much higher ratio than this, because the level of RC per unit of labor will also be low at the start of the development trajectory.

Parameter assumptions are selected to reflect reality in various ways. First, they imply that physical capital accounts for 30% of output, production labor 45% and RC 25%. This measure of the importance of RC is con-

---

Footnote:

7 This $\varphi$ is so high that no creative destruction has taken place before the start of any scenario, i.e. the starting situation is the same as the steady state situation of having $\varphi = \infty$. 

14
servative. In a pioneering study, Machlup (1962) estimated the share of all economic activity in the United States devoted to discovering and distributing information at 29%. Porat (1977) put it close to 50%. Second, values for $\lambda_t = \lambda$ and $g_0$ are sufficiently high for any economy to be able to catch up with the technological frontier within two decades if it invested all its resources (hence forgoing all output today, which is obviously not realistic) into technological progress via RC replacement. Third, parameter values reflect standard assumptions about discount rates (6% a year), saving rates (30% a year), and the rate of technological progress (2% a year). There remains arbitrariness especially with respect to $\varphi_t$ and $\lambda_t$. We discuss robustness of our results in the last subsection of the simulations.

In many models of development, it is difficult to capture the notion of systemic change. The two parameters in our model that capture systemic change are $\varphi_t$ and $\lambda_t$. A ‘big-bang’ systemic change can be represented as a one-off unanticipated change in $\varphi_t$ and/or $\lambda_t$. A continuous ‘improving’ systemic change is one where $\varphi_t$ and $\lambda_t$ continuously change, presumably in the direction of perfect markets, i.e. low $\varphi_t$ and high $\lambda_t$. Endogenous systemic change is one where $\varphi_t$ and $\lambda_t$ themselves are endogenous. In order to organize the discussion, we will simulate various scenarios.

4.3 Scenario 1: a transition

Scenario 1 is the development path of an economy that was initially characterized by the steady state of high $\varphi_t$ and a (low) $\lambda_t$, where overnight all political control is removed. There are no costs of breaking contacts so that $\varphi_t = \varphi = 1$ while also the labor costs of matching $\lambda_t = \lambda$ remain constant over time. In addition to this laisser-faire development path, we also show the theoretically optimal path a social planner would choose. This serves as a benchmark of what an optimal policy may accomplish.

Scenario 1 is apt for describing some event - a systemic collapse, a coup, a sudden policy change - that ends economic control over the economy. The outstanding example would be the post-socialist transition countries, with
sudden and comprehensive introduction of liberalizing policy measures. We assume throughout that firms maximize discounted-profits and have rational expectations after the shock. We contrast the outcome of their behavior with what the optimal solution would be that a benchmark all-knowing social planner would implement.

Concretely, we assume that at $t=0$, $\varphi_t$ suddenly changes from 1000 (virtually total political control) to 1 (no political interference at all), whilst nothing else changes and $\lambda_t = \lambda$ remains constant. Figures 4a and 4b show the simulation results for a decentralized transition; Figures 4c and 4d depict the 'optimal' path.
Figure 4a: "the optimal" capitalist transition

Figure 4b: relational capital investments and contact rates during the 'optimal' capitalist transition

Figure 4c: "the optimal" development path
The decentralized development path is characterized by a large initial decline in output, sustained over several periods. The decline in output in the first 7 periods is about 50%, which is mainly due to the reduction in RC and partly due to labor used in creative destruction. Output returns to the initial output level only after 20 periods. These figures qualitatively mimic the real patterns of output fluctuations in formerly centrally planned economies. The start of reform led in all 27 transition countries to a fall in output during three to eight years, a fall ‘never before experienced in the history of capitalist economies (at least in peacetime)’ (Mundell, 1997; see EBRD, 2003 for figures). More generally, Greenaway et al (2002), survey the experience of 25 developing countries which implemented ‘deep’ market liberalization programmes. In a panel data analysis, they demonstrate that market liberalization is typically followed by a J-curve output response over time: output falls steeply initially and recovers afterwards. More recently, Indonesia after the fall of Suharto and his network in 1998 exhibited a similar response.

For other parameter choices too, we find that the sudden drop in $\varphi_t$ without a change in $\lambda_t$, i.e. the advent of laisser-faire capitalism, destroys much of the existing networks in the economy. The reason is that the new

---

8We searched amongst the grid defined by $\gamma_0 \in \{0.5,0.65,0.8\}$, $\gamma \in \{0.6,0.7\}$, $g_0 \in \{0.2,0.5,1\}$, $g_1 \in \{0.5,1.5,4\}$, $\lambda(.) \in \{0.2,0.4,0.8\}$, $\beta = \{1,5\}$, $y(.) \in \{\text{Cobb-Douglas,CES}\}$.
system inherits a large network and backward technology. Maximizing firms have an incentive to upgrade their technology via high $D^R_t$, which rapidly destructs old networks.

Beyond the evidence on transition and developing countries quoted, another empirically verifiable implication of this model is that the lifting of barriers to creative destruction should lead to high demand for labor involved in networking, i.e. $L^R_t$, as opposed to production work. This should be observable as swift changes in rewards for making contacts. Such an immediate change has indeed been documented for Slovenia (Orazem and Vodopivec 1997), Russia (Brainerd, 1998; Sabirianova and Sabirianova, 2003) the Czech Republic (Flanagan, 1998) and China (Lee, 1999). These demonstrate that the returns to management skills, and more generally the skill wage premium, rose quickly and immediately after the start of the institutional changes.

The negative effects of high levels of creative destruction on the total level of $RC$ in the first periods generate a strong contraction in $y_t$. Because of complementarities, it is accompanied by a reduction in the marginal value of other production factors labor and capital. This concurs with observed increasing incidences of poverty and capital flight after market liberalization measures, of which the post-socialist transition is again an extreme example.

After liberalization, productivity would increase in the surviving firms due to the creative destruction they implement. Pavcnik (2000), using plant-level panel data on Chilean manufacturers, finds evidence of within plant productivity improvements following the Chilean liberalization of the early 1980s. She attributes this to ‘the reshuffling of resources and output from less to more efficient producers’. Similarly, Lall (1999) researches the garment industry in Kenya, Tanzania, and Zimbabwe, based on firm-level data, and finds technology upgrading and improving firm performance in response to liberalization. Grant (2001) similarly reports reallocation of enterprise relations in Ghana after reforms. In particular, his analysis points to increasing service-sector performance. Abandoning local control in particular led to rapid re-alignments in Ghana, with foreign companies establishing joint ven-
tures, developing local products, and joining national stock markets. These are indications that constituent firms were changing their production processes, their input suppliers and their clients. This may be interpreted as evidence of much contact replacement $D_t$.

We now turn to the optimal development path, i.e. the path of a social planner who would take the externalities of creative destruction into account. In Figure 4c, the super-planner chooses $D_t^c$ such that there is an initial output fall of about 30%. The initial levels of creative destruction are about 30% of that of the decentralized transition. The economy recovers to its old level after 10 periods, with high growth levels recorded in the early years. Growth in this period is fuelled by growth in the technology used. As in the earlier simulation, output growth eventually tails off to the level of exogenous progress of the technological frontier.

The interesting question is how any realistic policy can mimic the super-planner solution. The dilemma is that in practice no planner can engage in creative destruction since this requires decentralized information; but decentralized creative destruction overshoots. An observed policy is a dual track approach. In the case of China some restrictions on the mobility of labor and capital are maintained (Tian, 1999). As Roland and Verdier (2003) comment, such "...dualism follows the scenario of Chinese transition where the government keeps direct control over economic resources and where a liberalized non-state sector follows market rules". In terms of our model, the Chinese experience is a way to restrict the actions of a sizeable proportion of the firms in the economy, allowing only a fraction to engage in creative destruction, hence avoiding a cumulation of the external effects.9

The simulations above suggested that our model is capable of capturing observed economic dynamics after a momentous liberalization. Obviously, the speed of recovery varies tremendously with parameter variations, but the

---

9Additionaly, after the reform often local party members obtained the means of production form state companies (Lin, 2001). This realigns incentives and implies in our model a reduction of $\varphi$. Lee (1999) shows that these companies experience high growth rates.
qualitative finding of an output drop caused by a collapse of RC followed by a recovery appeared in all parameter values examined.

4.4 Scenario 2: gradual but inevitable system changes

In Scenario 1, it was effectively presumed that political institutions changed suddenly and completely, whilst there was no change in the rate at which individuals could make contacts. For many developing countries, it would seem more apt to assume that both political barriers and contact rates move slowly towards perfect markets. We leave the question of the endogeneity of such changes till the next subsection and here take them as inevitable.

Scenario 2 is the development path of an economy that was also initially characterized by the steady state of a high $\phi_t$ and a low $\lambda_t$, which sets upon a trajectory of ever decreasing $\phi_t$ and ever increasing $\lambda_t$. Letting $\phi_t$ decrease represents a gradual development of Market Institutions which lower contact matching costs, while simultaneously labor costs of contacting are falling. Again, not only the actual development path, but also the theoretically optimal path is shown.

More precisely, starting from the same conditions as above, we assume that from $t = 0$ onwards $\phi_t = 1 + \phi_0 e^{-\alpha \phi_{st}}$ and $\lambda_t = \lambda_3 * (1 - e^{-\lambda_2 - \lambda_4 st})$. This describes slowly adjusting $\phi_t$ and $\lambda_t$. We take $\phi_0 = 1000$, $\alpha_\phi = 0.05$, $\lambda_4 = 0.01$, $\lambda_2 = 0.05$ and $\lambda_3 * (1 - e^{-\lambda_2}) = \lambda_0$. These assumptions mean we allow $\phi_t$ to halve its distance towards 1 about every 8 years, and $\lambda_t$ to halve its distance towards $\lambda_3$ every 40 years. We show simulations with different choices later.
Again we find a sharp decrease in RC with the decentralized path. It is interesting that the optimal path includes maintaining RC for the first 20 years, illustrating the large negative externality of creative destruction on growth.

We here leave aside the actual composition of the increases in community social capital and market institutions (captured by a growing $\lambda_t$). In practice, market institutions may well replace Community Social Capital due to increasing returns to scale. Case studies document such substitution in banking (Ferrary 2003) and legal systems in the case of China (Winn 2002).
4.5 Scenario 3: endogeneity of system change

Scenario 3 is the development path of an economy that was initially characterized by the steady state of a high \( \varphi_t \) and a low \( \lambda_t \), which sets upon an trajectory of endogenous change in \( \varphi_t \) and \( \lambda_t \). Here we simulate the assumptions - introduced in section 3 - that the larger the market network (reflected in the value of the \( RC_t \) stock), the smaller \( \varphi_t \) and \( \lambda_t \), and that more \( RC \) leads to reduced political barriers to creative destruction. As we noted, the feed-back can lead to cyclical behavior in creative destruction and \( \varphi_t \).

We model this endogeneity by taking \( \varphi_t = 1 + \varphi_0 e^{-\beta_\varphi RC_{t-1}} \) and \( \lambda_t = \lambda_5 \ln(e + RC_{t-1}) \ln(e + \bar{y}_{t-1}) \) where \( \beta_\varphi = 0.4 \), and \( \lambda_5 \ln(e + RC_{t-1}) \ln(e + \bar{y}_{t-1}) = \lambda_0 \) which means \( \lambda_5 = 0.2846 \). Again, we will vary these assumptions later.

The simulations presented in figures 6a and 6b result.
Note the political cycles in the decentralized case, where only after 50 years the economy escapes the trap noted above. ¹⁰ Note also that the optimal development path first entails a period in which the RC network is expanded until $\phi$ is very low, i.e. first the political influence of politicians on the economy is removed. Only after that does the economy follow a path reminiscent of the decentralized path.

¹⁰Political cycles and the frequent un-doing of reforms after elections is, according to the historical analysis of Block (2002), a frequent phenomenon in African countries.
4.6 Robustness analysis

We had some empirics to guide us with respect to basic economic parameter assumptions. Yet there is simply nothing as yet to base $\lambda_t$ and $\varphi_t$ upon. For this reason, we give below the decentralized results for alternative assumptions (Figure 7). The main point we take from this is that results change commensurately with changes in the key parameters.

In the second endogenous simulation, for instance, the growth trap due to political institutions is so deep, and the contact rates so low, that even after 200 periods, the economy has not yet realized fast growth (average growth is less than 1.5% a year in this period). In the fourth endogenous growth path, the political growth trap is so small that the economy virtually immediately starts catching up and enters the steady state growth path after about 60 years.

In the first three exogenous growth paths, we see qualitatively similar growth paths to the one in Figure 5, i.e. initial decades of very low RC due to initial creative destruction. Only after 20 years does the growth in $\lambda_t$ allow the economy to achieve high growth levels. Interestingly, in the exogenous simulations where the political reform is slower ($\alpha_\varphi$ is low in simulations 4 and 5), the initial collapse of RC does not occur and sustained growth appears almost immediately.

This dependence of development paths on parameter choices reflects the importance of initial conditions but also the importance of contact rates - depending, in turn, on market institutions and Community Social Capital - and the level of political interference with the market.
Figure 7: development paths under different parameter values

Exogenous systemic growth:  Endogenous systemic growth:

1. \( \alpha = 0.01, \quad \beta = 0.8; \) With 2.: \( \alpha = 0.1, \quad \lambda_2 = 0.1, \quad \beta = 0.2. \)

With 3.: \( \alpha = 0.1, \quad \lambda_2 = 0.1, \quad \lambda_4 = 0.02, \quad \beta = 0.2, \quad \lambda_5 = 2 \times 0.2846 \)

With 4.: \( \alpha = 0.025, \quad \lambda_2 = 0.1, \quad \lambda_4 = 0.02, \quad \beta = 0.4, \quad \lambda_5 = 2 \times 0.2846 \)

With 5.: \( \alpha = 0.025, \quad \lambda_2 = 0.1, \quad \lambda_4 = 0.005, \quad \beta = 0.8, \quad \lambda_5 = 2 \times 0.2846 \)
5 Conclusions

In this paper, we introduced the notion of Relational Capital in an endogenous growth model. RC represents the stock of contacts of individuals in an economy. RC is an input into sold output. Both Community Social Capital - which we define as the size of the informal network - and Market Institutions reduce the labour costs of creating RC. Political interference in our model increases the costs of breaking up contacts among firms. We argue that this breaking up of contacts is an integral part of technological advancement. If the political process restricts such creative destruction by raising its costs, technological backwardness results.

Economic systems with bureaucratic interference through corruption, nepotism or planning lag behind in the level of technology employed according to our approach. These economies are likely to experience an initial output fall if they liberalize: technological catch-up potential implies high initial levels of destroyed and replaced relational capital which incorporates a large negative external effect. In the simulations such drops indeed occurred endogenously from the optimising behaviour of rational firms. Our model leads to support for smoother reforms such as ‘dual track’ approaches discussed in the literature. One policy instrument to restrict some of the externalities created by creative destruction is to have complete systemic change only in new sectors of the economy.

The simulations suggested another interesting empirical implication. With endogenous feedbacks from the size of the economy to the costs of replacing and making contacts, we find cycles. These quasi business cycles point to a coordination phenomenon. When an economy is close to the technology frontier, investment in new contacts is more productive than replacing a contact. Once the economy is far from the frontier the opposite is true. Coordination of activity follows from the observation that new contacts live longer if most of the economy refrains from replacing contacts, hence the relative cost of replacing a contact is high. Vice versa, new contacts have a low survival rate if the economy engages heavily in replacing contacts. In this situation replac-
ing a contact is relatively cheap. A full dynamic analysis of these endogenous cycles constitutes an interesting extension of our analysis.

There are various avenues that can be pursued further. One is the precise nature of political interaction. The political system was implicitly defined in our model as a function of the total stock of RC. The feedback from large networks to less political frustration of the replacement of contacts needs a further foundation. In Dulleck and Frijters (2003) we study how and to what extent those in power frustrate the growth of relational capital, simply because it poses a political threat to their power.

Another avenue for further investigation centers around the parameter $\beta$. We assume that the complexity of production is exogenous to the model and time-invariant. In our model $\beta$ measures the length of a production chain as a proxy for such complexity. It determines the number of firms that are affected by the creative destruction of one element in the chain. A further step in the analysis would be to endogenize $\beta$. The endogeneity of this parameter may capture the development of productivity in relation to the division of labor. Empirical observations by Hedlund and Sundstrom (1996) show that liberalization mostly affects those firms with the highest value-added, which usually have the most complex production processes. The ‘primitivization’ of transitional economies can be seen as an endogenous reduction of $\beta$.

This set of applications shows the potential of our framework. We offer it as one way to theoretically connect the literatures on networks to those on politics and economic growth.
References


Appendix 1: a search model of relational capital.

Appendix 1.1 The basic model of RC

In this appendix we motivate the macro-model of creative destruction by a micro-search model. We will borrow arguments from the search literature by exploiting the analogy with the matching process of vacancies and job-seekers (Petrongolo and Pissarides, 2001).

Denote the number of contacts a representative individual firm $i$ has by $C_i$. Denote the number of extra contacts a firm makes by $N_i$ and the number of contacts it replaces by $D_i$. Take the number of firms $M$ to be large, such that the proportion of contacts any firms has is approximately zero. When firm $i$ replaces an old contact with a new one, it loses a previous contact. The firm $j$ with whom firm $i$ makes a replacement contact also loses a previous contact. Hence both firm $i$ and $j$ remain with the same number of contacts as before. The externality is that the two firms that $i$ and $j$ were previously connected to, lose a contact. If these former contacts were necessary links in a network of $k$ contacts, the net loss of contacts is $\beta = 2k - 1$. The number of existing, new, and destroyed contacts is assumed large enough to be able to abstract from indivisibilities.

The timing is as follows. At the beginning of the period, firms seek extra contacts and replacement contacts. Then, these latent contacts materialize, after which production takes place. Then, the technology to be used next period is updated.

The probability of any contact surviving the process of creative destruction is equal to $(1 - \frac{1}{\sum_i C_i})^{\sum_{j \neq i} \beta D_j}$ which is in the limit ($M \to \infty$) equal to $e^{-\beta \bar{D} / \bar{C}}$. The number of contacts of firm $i$ after creative destruction and extra contacts is equal to $C_i * e^{-\beta \bar{D} / \bar{C}} + N_i$. Adding time subscripts and re-labelling, this is the same as the formula for $RC_t$ given in the main text. Note that here the replacement contacts are treated as cumulative, i.e. it is possible to replace the same initial contact several times in one period, leading to a larger technological improvement. In contrast, extra contacts are additive.
Appendix 1.2  Modelling the endogeneity of contact rates

We can similarly give a micro-foundation for $\lambda(.)$, i.e. the relation between labor invested into making new contacts, the number of old contacts and the number of new (extra and replacement) contacts. We again exploit the analogy with job search. We thus envisage the process of finding contacts as follows: denote the amount of labor firm $i$ allocates towards creating extra contacts by $L_{N,i}$ and the amount allocated towards replacing contacts by $L_{D,i}$. This labor is directly and linearly transformed into ‘active contact vacancies’ whereby the old contacts involved in replacements are only actually destroyed if a partner for the replacement contact is found. We can hence also use $(L_{N,i} + L_{D,i})$ to denote the number of contact vacancies firm $i$ has. We then have a symmetric matching situation whereby $L_{N,i}$ number of potential contacts of each firm get matched to the $\sum_{j\neq i} L_{N,j}$ potential extra contacts of other firms. The total amount of extra contacts can then be represented by a matching function $m(\sum_{j\neq i} L_{N,j}, \sum_{j\neq i} L_{N,j})$. As Petrongolo and Pissarides (2001) show, there are several micro-mechanisms via which we can arrive at a linear matching function, implying that the total number of extra contacts is linear in the number of potential extra contacts. One such possible mechanism is that each individual latent contact has a fixed probability $\lambda$ of being ‘noticed’, which is a ‘fixed advertisement space’ assumption. All these ‘noticed’ latent contacts then get randomly matched to each other. This then indeed would imply a constant returns to scale matching function and a linear relation between the amount of labor devoted to making extra and replacement contacts and the number of new extra and replacement contacts.

The political process can now be summarised by the assumption that politicians allow a contact replacement to go ahead with probability $\frac{1}{\phi_t}$. Together with the above, this means we get $\lambda_t*(L_{N,it} + L_{D,it})=\phi_tD_{it} + N_{it}$, which is the same formula as the one in the text.

Now, we can also endogenize $\lambda$ in a way that links it to the number of contacts already existing in the economy. A natural possibility is to assume
that it is the two sides of an ‘old’ contact via which latent contacts get noticed. Assume for instance that there is a constant probability that a latent match is productive termed $\lambda_0$. The probability that a latent contact is observed by an existing contact is infinitesimally small and denoted by $\lambda_1$. The probability that an individual latent contact gets labelled as a ‘noticed and productive’ contact is then equal to $\lambda_0 \times (1 - (1 - \lambda_1)\sum_{j \neq i} C_j)$ which converges to $\lambda_0 \times (1 - e^{-\lambda_1MC})$. In terms of the formulas in the text, this would mean the function $\lambda(RC_{t-1}) = \lambda_0 \times (1 - e^{-\lambda_1RC_{t-1}})$ is a natural candidate which has the standard convexity properties. Various other micro-mechanisms leading to such relations also exist however. The key aspect is that the thick-market externality of Diamond (1982) is incorporated. In the example above, this thick-market externality is incorporated in the assumption that each side of an existing contact has an independent probability of noticing a latent contact. This is a network externality of having many existing contacts.

Appendix 1.3  Foundation of the process of technological change

Finally, we can think of the following stylized micro-foundation to our process of technological change. Take each representative firm to consist of a fixed number of labour units, say $Z$ units. The technology used by each labour unit $i$ depends on one contact (eg. the machine provider or the service department of another firm). Different units in the same firm may or may not use the same contact as the technology source. Each labour unit $i$ then combines the other contacts and capital to produce sold output. Economies of scale ensure that at the firm level $y_t$ increases with $RC_t$. Now, the technology of the match between unit $i$ and her contact is on average $A_{t-1}$. The firm can search for more contacts ($N_t$) and/or to find different technology contacts ($D_t$). If a unit $i$ changes a technology contact, her previous technology contact becomes redundant because economies of scale in doing any specific task make the productivity of unit $i$ highest when working only with one technology contact (eg. using one word processing program is more efficient that working with two simultaneously). The firm observes two equally sized sets of candidate contacts it can search from, one for $D_t$ and one for $N_t$. 38
The equal size assumption means the symmetry assumed in the matching stories above between $D_t$ and $N_t$ remains valid, and the previous matching arguments go through after appropriate normalisation. The distribution of technical productivity of potential ‘different’ contacts is in continuous flux: every period, the productivity that unit $i$ would have with a different technology contact $j$ is drawn from a c.d.f. $H_t(.)$, where $H_t(A_{t-1}) = 0$ and $H_t(A_{t-1} + g_o(A^*_t - A_{t-1})) = 1$. This means a firm can observe ‘a region of potential better matches’ that lie within a fraction $g_o$ between the productivity of a current match and the technological frontier. One can think of $H_t(.)$ as the result of an exogenous, random, and continuous learning process that other potential matches undergo whilst they are inactive. The expected technical productivity of the ‘different’ technology contacts would thus be $A_{t-1} + E_t[H_t(.) - A_{t-1}](A^*_t - A_{t-1})$. Within one period, the process of finding a different set of matches starting from the current (potentially latent) technology can be repeated many times in the same period until the eventual set of contacts is finally effectuated and the old ones are severed. If $g_0$ is small, then the expected result of one period of technological change goes to $A_{t-1} + (1 - e^{-g_0})(A^*_t - A_{t-1})$ where $g_0 = M_tE_t[H_t(.) - A_{t-1}]/(A^*_t - A_{t-1})$ and $M_t$ is the number of ‘rounds of innovation’ per labour unit in the period. When $M_t$ is reasonably small, the probability of any contact surviving the contact destruction by other firms will approach $e^{-\beta \bar{D}}$.

If we add an exogenous probability $(1 - g_1)$ that the firm is completely mistaken about each unit’s set of potential new technology contacts (where the mistake is revealed only after all rounds of innovation), and relate $M_t$ to $D_t$, then we get the technological progress function specified in the simulations.

Appendix 2: Steady states and simulation technique.

In this appendix we describe our approach to deriving steady states and computing the transition paths. Our general approach is to find the steady
state from the Euler equations and then to solve the transition path by presuming the steady state is reached at some fixed date into the far future and solving an optimal control problem backwards. Though we simulate various models, we here merely illustrate our method by taking a simplified version of the model. We presume interior solutions (and thus ignore non-negativity constraints), take the case where $\lambda$ and $\varphi$ are fixed, and hold labor fixed and drop capital from the model. Also define $\mu_t = \frac{A_t}{A_{t-1}}$. This means the maximization problem becomes

$$\max_{D^c_t, N^c_t} U_s = \sum_{t=s}^{\infty} e^{-\rho(t-s)} \mu_t A^*_0 (1 + a)^{t-s} (L - L^c_t)^{\gamma_0} RC_t^{1-\gamma_0}$$  \hspace{1cm} (6)$$

subject to:

$$\lambda L^c_t = \varphi D^c_t + N^c_t$$  \hspace{1cm} (7)$$
$$\mu_t = \frac{\mu_{t-1}}{1 + a} + g_t (1 - e^{-g_0 D_{t-1}}) (1 - \frac{\mu_{t-1}}{1 + a})$$  \hspace{1cm} (8)$$
$$RC_t = RC_{t-1} e^{-\beta \frac{\bar{D}^c_t}{\mu_{t-1}}} + N^c_t$$  \hspace{1cm} (9)$$
$$A^*_t = (1 + a) A^*_{t-1}$$  \hspace{1cm} (10)$$

The 2 state variables in this model are then $\mu_{t+1}$ and $RC_t$. First, we solve out all the non-state variables. We then get

$$\max_{\mu_{t+1}, RC_t} U_s = \sum_{t=s}^{\infty} e^{-\rho(t-s)} \mu_t A^*_0 (1 + a)^{t-s} (L^P)^{\gamma_0} RC_t^{1-\gamma_0}$$  \hspace{1cm} (11)$$

where $L^P = L - \frac{\varphi}{g_0} \ln (1 - \frac{\mu_{t+1} - \mu_t}{g_t (1 - \frac{\mu_t}{1 + a})}) + RC_t - RC_{t-1} e^{-\beta \frac{\bar{D}^c_t}{\mu_{t-1}}}$$  \hspace{1cm} (12)$$
$$\frac{-1}{g_0} \ln (1 - \frac{\mu_{t+1} - \mu_t}{g_t (1 - \frac{\mu_t}{1 + a})}) = D_t \text{ and } N^c_t = RC_t - RC_{t-1} e^{-\beta \frac{\bar{D}^c_t}{\mu_{t-1}}}$$  \hspace{1cm} (13)$$

where the decision maker takes $\frac{\bar{D}^c_t}{\mu_{t-1}}$ as given. The first-order conditions are then
which uses the envelope theorem: for all other state variables \( z_t \notin x_s = \{\mu_s, RC_s\} \) we know that \( \frac{\partial U}{\partial z_t} = 0 \) because in an optimum \( \frac{\partial U}{\partial z_s} = 0 \). Inserting gives

\[
\frac{y_s}{\mu_s} + \frac{-\varphi}{\lambda} D'_s(\mu_{s+1}) \frac{\partial y_s}{\partial L} + e^{-\rho} \frac{-\varphi}{\lambda} D'_{s+1}(\mu_s) \frac{\partial y_{s+1}}{\partial L} = 0
\]

\[
(1 - \gamma_0) \frac{y_s}{RC_s} + \frac{-1}{\lambda} \frac{\partial y_s}{\partial L} + e^{-\rho} \frac{-\varphi}{\lambda} \frac{1}{\partial L} e^{-\beta} \frac{D_{rc}^{s+1}}{\mu_s} = 0
\]

\[
\frac{1}{g_0(g_1(1 - \frac{\mu_s}{1+\alpha}) - \mu_{s+1} + \mu_s)} = D'_s(\mu_{s+1})
\]

\[
\frac{1}{g_0(\mu_{s+2} - g_1) (1 + \alpha) + \mu_{s+1} (g_1 - 1)} (1 + a - \mu_{s+1}) = D'_{s+1}(\mu_{s+1})
\]

where we have used terms like \( D'_s(\mu_{s+1}) \) and \( D'_{s+1}(\mu_{s+1}) \) to denote specified complex functions of the state variables. Now, a rational expectations equilibrium must have \( RC_s = RC_s^c \) and \( D_{rc}^s = D_{rc}^s^c \). Thus inserting the relevant function for \( D_{rc}^{s+1} \) gives

\[
\frac{y_s}{\mu_s} + \frac{-\varphi}{\lambda} D'_s(\mu_{s+1}) \frac{\partial y_s}{\partial L} + e^{-\rho} \frac{-\varphi}{\lambda} D'_{s+1}(\mu_s) \frac{\partial y_{s+1}}{\partial L} = 0
\]

\[
(1 - \gamma_0) \frac{y_s}{RC_s} + \frac{-1}{\lambda} \frac{\partial y_s}{\partial L} + e^{-\rho} \frac{-\varphi}{\lambda} \frac{1}{\partial L} e^{-\beta} \frac{D_{rc}^{s+1}}{\mu_s} = 0
\]

\[
\frac{1}{g_0(g_1(1 - \frac{\mu_s}{1+\alpha}) - \mu_{s+1} + \mu_s)} = D'_s(\mu_{s+1})
\]

\[
\frac{1}{g_0(\mu_{s+2} - g_1) (1 + \alpha) + \mu_{s+1} (g_1 - 1)} (1 + a - \mu_{s+1}) = D'_{s+1}(\mu_{s+1})
\]

We now divide both lines by \( y_s \) and write out. We then get
\[
0 = \frac{1}{\mu_s} + \frac{\varphi}{\lambda} D'_s(\mu_{s+1}) \gamma_0
\]

\[
* \quad (L - \frac{\varphi}{g_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)}) + RC_s - RC_{s-1} e^{-\frac{1}{\gamma_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)})})^{-1}
\]

\[
+ \quad e^{-\rho} \frac{\varphi}{\lambda} D'_{s+1}(\mu_{s+1}) \gamma_0 (1 + a) \mu_{s+1}
\]

\[
* \quad (L - \frac{\varphi}{g_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)}) + RC_{s+1} - RC_{s-1} e^{-\frac{1}{\gamma_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)})})^{-1}
\]

\[
0 = \frac{(1 - \gamma_0)}{RC_s} + \frac{1}{\lambda} \gamma_0
\]

\[
* \quad (L - \frac{\varphi}{g_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)}) + RC_s - RC_{s-1} e^{-\frac{1}{\gamma_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)})})^{-1}
\]

\[
+ \quad e^{-\rho} \frac{1}{\lambda} \gamma_0 (1 + a) \mu_{s+1} (L - \frac{\varphi}{g_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)}) + RC_{s+1} - RC_{s-1} e^{-\frac{1}{\gamma_0} \ln(1 - \frac{\mu_{s+1} - \mu_t}{g_1(1 - \mu_t)})})^{-1}
\]

\[
* \quad \frac{RC_{s+1}^{-\gamma_0} e^{-\rho}}{RC_{s-1}^{\gamma_0}} = \frac{1}{g_0 (g_1 (1 - \frac{\mu_{s+1}}{1 + a}) - \mu_s + \mu_{s-1} \frac{1}{1 + a})}
\]

\[
D'_s(\mu_{s+1}) = \frac{1}{g_0 (g_1 (1 - \frac{\mu_{s+1}}{1 + a}) - \mu_s + \mu_{s-1} \frac{1}{1 + a})}
\]

\[
D'_{s+1}(\mu_{s+1}) = \frac{(1 - \mu_{s+1}) (1 + a)}{g_0 \left[ (\mu_{s+1} - g_1) (1 + a) + \mu_s (g_1 - 1) \right] (1 + a - \mu_s)}
\]

We proceed to find a stationary growth path. This implies setting \( RC_s = RC_{s+1} = RC \), \( y_{s+1} = (1 + a) y_s \), and \( \mu_{s+1} = \mu_s = \mu \). Inserting gives
\[ 0 = \frac{1}{\mu} + \left[ -\frac{\varphi}{\lambda} D_1 \gamma_0 + e^{-\rho} \frac{\varphi}{\lambda} D_2 \gamma_0 (1 + a) \right] \]

\[ \ast (L - \frac{\varphi^{-1} \ln(1 - \frac{a\mu}{g_1(1-(1-a)\mu)})}{\lambda} + RC - RC e^{-\beta \frac{\varphi^{-1} \ln(1 - \frac{a\mu}{g_1(1-(1-a)\mu)})}{RC}}) \]^{-1} \]

\[ 0 = \frac{(1 - \gamma_0)}{RC} + \left[ \frac{-1}{\lambda} \gamma_0 + e^{-\rho} \frac{1}{\lambda} \gamma_0 (1 + a)e^{-\beta \frac{\varphi^{-1} \ln(1 - \frac{a\mu}{g_1(1-(1-a)\mu)})}{RC}} \right] \]

\[ \ast (L - \frac{\varphi^{-1} \ln(1 - \frac{a\mu}{g_1(1-(1-a)\mu)})}{\lambda} + RC - RC e^{-\beta \frac{\varphi^{-1} \ln(1 - \frac{a\mu}{g_1(1-(1-a)\mu)})}{RC}}) \]^{-1} \]

\[ D_1 = \frac{1}{g_0 (g_1 (1 - \mu) + (g_1 - 1) a \mu)} \]

\[ D_2 = \frac{(1 - \mu) (1 + a)}{g_0 [-g_1 (1 + a) + \mu (g_1 + a)] (1 + a - \mu)} \]

which means we have finally arrived at a 2-equation system in which the only unknowns are \( \mu \) and \( RC \). Unfortunately, the functions are analytically intractable. It is however quite simple for specific values of all parameters to solve this system numerically. This would yield a set of rational expectation stationary steady state values \( \{ \mu, RC \} \).

The procedure we adopt to calculate the simulations is the following:

1. We go through the steps above to reach an analytical equation for the model and parameters at hand.

2. We numerically solve for all candidates for a stationary steady state and pick the one that satisfies second-order conditions (we never find more than 1 of these).

3. We presume the steady state is reached at some date \( s \) far in the future (usually 200 years).

4. We then calculate the transition path decisions \( \{ \mu_{t+1}, RC_t \} \) for \( t = 0, \ldots, s - 1 \).
Now, calculating the transition path decisions is a tricky optimal-control problem by itself. A popular method in the literature is to take the Euler equations and either guess backward or forwards. To illustrate, if one uses forward-shooting one takes as given the initial condition \( \{ \mu_0, RC_{-1} \} \), and one then picks an initial guess \( \{ \mu_1, RC_0 \} \). With this initial guess, one uses the Euler equations to solve for all other \( \{ \mu_t, RC_t \} \) with \( t = 1, \ldots, s - 1 \). We can then check whether the initial guess is correct by looking at whether the inferred \( \{ \mu_s, RC_{s-1} \} \) satisfies the stationary steady state assumptions

\[
RC_s = RC_{s-1}, \quad y_s = (1 + a)y_{s-1}, \quad \text{and} \quad \mu_s = \mu_{s-1}.
\]

If we then find that these are not satisfied, one needs to take a different initial guess \( \{ \mu_1, RC_0 \} \).

In our case we have to take account of the fact that \( N_t \) and \( D_t \) are constrained to be positive. Indeed, in many simulation paths these constraints on \( N_t \) and \( D_t \) turn out to be binding in many periods. The ‘correct’ procedure would be to check all possible combinations of \( N_t \) or \( D_t \) being 0 at all possible times \( t = 1, \ldots, s - 1 \). When applied to forward-shooting, this would mean that for each initial guess \( \{ \mu_1, RC_0 \} \), one would have to check \( 2^{2s} \) combinations of constraints. For \( s = 200 \), this would mean checking \( 2.5822 \times 10^{120} \) combinations. It will be clear that such an avenue is infeasible. We therefore use an approximating heuristic. What we in practice do is to set

\[
\frac{N_t}{L\lambda_t} = \frac{e^{a_1 t}}{1 + e^{a_1 t} + e^{a_2 t}},
\]

\[
\frac{\varphi D_t}{L\lambda_t} = \frac{e^{a_2 t}}{1 + e^{a_2 t} + e^{a_2 t}},
\]

\[
L - L_t^{ec} = \frac{1}{1 + e^{a_1 t} + e^{a_2 t}}
\]

where the decisions of the representative agent now are \( a_1 t \) and \( a_2 t \), and where we denote the aggregate choice by \( \{ a_1 t, a_2 t \} \). This transformation prevents the possibility of corner solution but allows choices arbitrarily close to a corner solution: for very low \( a_1 t \), \( N_t \) gets arbitrarily close to 0. This approximation in turn however does not allow for forward shooting via the use of the Euler equations because in the case of a corner solution, the first-
order condition will be violated (i.e. no finite solution to $a_{1t}$ and $a_{2t}$). What this transformation does however allow is to be used as an input into a search routine for an optimal solution. Because the objective function ($=U_s$) is highly non-linear we use various search algorithms. A complication of not being able to use the Euler equations is that we must continuously bear in mind that the representative agent takes the aggregate choice as given. Our actual heuristic for step 4 is thus:

4a Start at an initial guess for the whole sequence $\{\bar{a}_{1t}, \bar{a}_{2t}\}$ where $t = 0, \ldots, s - 1$.

4b Use a non-linear optimisation routine to find the optimal choices of the individual agent for $\{a_{1t}, a_{2t}\}$ where $t = 0, \ldots, s - 1$, with the aggregate choices $\{\bar{a}_{1t}, \bar{a}_{2t}\}$ kept constant.

4c If the optimal choices of the individual agent for $\{a_{1t}, a_{2t}\}$ deviate by more than a very small amount from $\{\bar{a}_{1t}, \bar{a}_{2t}\}$, then step 4a is repeated with the aggregate choices re-set at $\{a_{1t}, a_{2t}\}$.

4d Given the optimal solution in terms of $\{a_{1t}, a_{2t}\}$, we calculate all other parameters of the model, including $\{\mu_t, RC_{t-1}\}$ where $t = 0, \ldots, s - 1$.

Now, given enough time, a non-linear optimisation routine should always find the individually optimal sequence $\{a_{1t}, a_{2t}\}$ given what the aggregate choice is and thus a sequence for which there holds that $\{a_{1t}, a_{2t}\}$ is optimal when $\{\bar{a}_{1t}, \bar{a}_{2t}\} = \{a_{1t}, a_{2t}\}$ should indeed be a Rational Expectations solution to the whole model.

We numerically checked the stability of the model using the Euler equations. This entailed linearising $\{\mu_{s-1}, \mu_s, \mu_{s+1}, RC_{s-2}, RC_{s-1}, RC_s\}$ around the steady state. This gives equations of the form
The stability of the equation \( S_{t+1} = B_0 S_t + B_1 S_{t-1} \) follows standard dynamic arguments. One examines the roots of the system solving the normal equation \( \det(\theta^2 I - \theta B_0 - B_1) \) which should give 4 values, \( \theta_1 \ldots \theta_4 \), i.e. 2 each for each state variable. One gets a stable stationary steady state when there are 2 roots within the unit circle and 2 outside. With one root within the unit circle, there is no stable stationary steady state. With 3 roots within the unit circle, indeterminancy follows. In our simulations, we found the number of unit roots to be in the unit circle to depend quite sensitively on parameter choices. The generic outcome was either 2 or 3 roots within the circle.