Abstract

This paper considers the size effect, where volatility dynamics are dependent upon the current level of volatility within an stochastic volatility framework. A non-linear filtering algorithm is proposed where the dynamics of the latent variable is conditioned on its current level. This allows for the estimation of a stochastic volatility model where dynamics are dependent on the level of volatility. Empirical results suggest that volatility dynamics are in fact influenced by the level of prevailing volatility. When volatility is relatively low (high), volatility is extremely (not) persistent with little (a great deal of) noise.

Keywords
Non-linear filtering, stochastic volatility, size effect, threshold

JEL Classification C13 C22 C53

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1 Introduction

Modeling the distribution of financial asset returns is a critically important issue. Application within areas such as risk management, portfolio construction (diversification) and option pricing require estimates of the distribution governing asset returns. To accurately capture the distribution of returns, it is necessary to capture time-variation of volatility. Therefore much research effort has been focused on estimating the condition volatility, or distribution of asset returns.

Estimating the conditional distribution of asset returns can be first attributed to the seminal work of Engle (1982) in developing the ARCH (AutoRegressive Conditional Heteroscedasticity) model, treating the distribution of returns as a normal distribution with time-varying variance. Bollerslev (1987) extended this framework to develop the GARCH model, and in doing so, led to the development of a wide range of models. Attempts to deal with features of returns such as asymmetries and excess kurtosis have been proposed by Nelson (1991). A vast amount of literature exists in this field, summaries of which are contained in Bollerslev, Chou and Kroner (1992), Pagan (1996) and Campbell, Lo and MacKinaly (1997).

An alternative approach to GARCH style models are the Stochastic Volatility (SV) class of models which treat conditional volatility as a latent variable that follows its own stochastic process. While on a practical level, it is difficult to estimate the parameters of SV models, theoretically they are appealing. Clark (1973), Tauchen and Pitts (1983) and Andersen (1996) theoretically motivate SV models from the perspective of capturing stochastic changes in information flow.

There has been an enormous amount of attention paid to the specification of volatility dynamics. Broadly speaking, the important features of volatility dynamics may be due to either sign (relationship with sign of past returns) and or size (relationship with size of past returns or volatility) effects. Much work has been directed at dealing with leverage in volatility, the asymmetric sign effect in
that negative returns lead to proportionally higher future volatility. Within the
GARCH class of models, Nelson (1991), Hentschel (1995) and Ding, Engle and
Granger (1993) and Gonzalez-Rivera (1998), among others, all propose various
threshold style models to capture the so called leverage effect. Within an SV
framework, Harvey and Shephard (1996) and So, Lam and Li (2002) propose
asymmetric SV models where volatility dynamics are dependant on the sign of
past returns, once again to capture the leverage effect. An alternative feature
that has received much less attention is that volatility dynamics may in fact be
dependant on the level of volatility. Friedman and Laibson (1989), Gouriéroux
and Monfort (1992), Engle and Ng (1993) and Longin (1997), within GARCH
style models, consider that volatility dynamics may in fact be dependant on
the level of volatility. For instance, Friedman and Laibson (1989) find that the
persistence in conditional volatility falls when shocks to US stock returns are
large.

To the best of the authors knowledge, the size effect in volatility has not been
considered within a SV framework. Partly, this may be due to the problems
surrounding the estimation of such models. It is therefore the goal of this paper
to develop a non-linear filtering algorithm that allows the dynamics of volatility
(the latent variable) in an SV setting to be dependent on the current level of
volatility. This non-linear filtering framework build upon the discretised non-
linear filter (DNF) approach to dealing with latent variable models proposed by
Clements, Hurn and White (2004). Given the non-linear filtering framework, an
hypothesis test is suggested to ascertain whether volatility dynamics are in fact
dependant on the level of volatility.

A related type of model to that proposed here would be a regime switching
SV model where the SV dynamics are dependant on the evolution of the state
of an unobserved Markov chain. If the states of the Markov process were to
reflect states of volatility (or levels) then such a regime switching model would
in some way capture the changing dynamics that were dependant on the level of
volatility. The threshold style model proposed in this paper, intuitively captures similar features of volatility dynamics without relying another unobserved state variable and regime switching framework.

This paper proceeds as follows. Section 2 introduces the concept of an SV model with size effects. Section 3 outlines the DNF estimation as it applies to a standard SV model, along with adjustments to capture the size effect. An approach to testing the significance of the size effect is also proposed. Section 4 present empirical application of the SV models with a size effect, showing this it is an importan feature of the two time series considered. Section 5 provides concluding remarks.

2 Stochastic volatility with size effects

A stochastic volatility (SV) model considers that returns (the observed variable) \( \{y_t\}_{t=1}^T \) are generated by,

\[
y_t = \exp(x_t/2) \, u_t \quad u_t \sim N(0, 1) \tag{1}
\]

where \( x_t = \ln(\sigma_t^2) \). SV models treat \( x_t \) as an unobserved (latent) variable, following its own stochastic path, the simplest being an AR(1) process,

\[
x_t = \alpha + \beta \, x_{t-1} + w_t \quad w_t \sim N(0, \sigma_w^2) \tag{2}
\]

where errors, \( u_t \) and \( w_t \) are assumed to be independent. These equations describe the standard SV model where volatility dynamics are independant of the current level of volatility.

To incorporate a size effect (or level dependence) into the SV dynamics it is necessary to condition the volatility dynamics on the level of volatility. To do so, the state-space of \( x_t \) will be partitioned by the point \( \tau \) into two adjoing regions each with their own distinct volatility dynamics. Two regions are selected in this context to reflect relatively high and low volatility. To allow for the size effect in
SV dynamics equation 2 must be augmented

\[ x_t = \alpha_s + \beta_s x_{t-1} + \sigma_{w,s} w_t, \]  

(3)

where the subscript \( s \) denotes the index of the region containing \( x_{t-1} \). If \( x_{t-1} > \tau \) dynamics will be governed by \( \theta_2 = (\alpha_2, \beta_2, \sigma_{w,2}) \) otherwise if \( x_{t-1} < \tau \) dynamics will be governed by \( \theta_1 = (\alpha_1, \beta_1, \sigma_{w,1}) \).

The following section will now outline the DNF estimation framework employed to estimate to the standard SV model of equation 2 along with an extension to deal with the level dependence implied by equation 3. The proposed hypothesis testing framework to determine the significance of the size effect is also discussed.

### 3 SV Estimation and Testing of size effect

To estimate the parameters of the SV model incorporating the size effect, this paper builds upon the non-linear filtering framework pioneered in Kitigawa (1987). While many other approaches to estimating SV models exist (for summaries see Ghysels et al. 1996 and Shephard 1996) the non-linear filtering approach has been chosen in this setting as it provides the flexibility required when incorporating non-standard features such as the size effect.

Within the non-linear filtering setting a number of estimation procedures have been proposed. Integration procedures for estimating the non-linear filtering equations have been proposed both by Watanabe (1999) and Fridman and Harris (1998). Simulation based filters which require bayesian estimation are provided by Kitigawa (1996) and Pitt and Shephard (1999). Whilst the latter would be amenable to incorporating the size effect, the associated computational cost is extremely high. Therefore to consider the size effect within an SV framework, the Discrete Non-Linear Filter (DNF) method of Clements, Hurn and White (?) is employed. The DNF approach is related to both Watanabe (1999) and Fridman and Harris (1998).
Estimation of a latent variable process such as equation 2 within a non-linear filtering framework is based on the recursive, prediction-update algorithm suggested by Kitagawa (1987). This approach requires two density functions to be defined and a number of integrals to be evaluated. Let \( r(y_t \mid x_t, \theta) \) be the conditional distribution of \( y_t \) on \( x_t \) (given equation 1), \( q(x_t \mid x_{t-1}, \theta) \) be the conditional distribution of \( x_t \) on \( x_{t-1} \) (given equation 2) and \( \theta = (\alpha, \beta, \sigma_w) \) in the standard SV case. The one-step ahead prediction of the distribution of \( x_t \) conditional on \( y_{t-1} \), \( f(x_t \mid y_{t-1}, \theta) \), is given by

\[
f(x_t \mid y_{t-1}, \theta) = \int_{-\infty}^{\infty} q(x_t \mid x_{t-1}, \theta) f(x_{t-1} \mid y_{t-1}, \theta) \, dx_{t-1}.
\]  

(4)

Once a new observation, \( y_t \), is available, the probability distribution of the state variable at time \( t \), conditional on information at time \( t \), \( f(x_t \mid y_t, \theta) \), may now be obtained as

\[
f(x_t \mid y_t, \theta) = \frac{r(y_t \mid x_t, y_{t-1}, \theta) f(x_t \mid y_{t-1}, \theta)}{f(y_t \mid y_{t-1}, \theta)}.
\]  

(5)

The denominator of equation (5) is the likelihood of observing \( y_t \) conditional on \( y_{t-1} \) and \( \theta \) and may be computed as

\[
f(y_t \mid y_{t-1}, \theta) = \int_{-\infty}^{\infty} r(y_t \mid x_t, \theta) f(x_t \mid y_{t-1}, \theta) \, dx_t
\]  

(6)

which may be optimised (for all observations) to permit maximum likelihood (ML) estimates of SV parameters to be obtained.

The subsequent section will discuss how the DNF may be applied to the estimation of the standard SV model of equation 2. This will be followed by a discussion of how both the non-linear filtering framework of equations 4 through 6 and the DNF method may be tailored to deal with the size effect. Finally an hypothesis testing approach for determining the significance of the size effect with the outlined.

3.1 Estimation of SV models using DNF

The DNF solves the non-linear filtering equations based on a discretisation of state-space. This allows the likelihood function of a continuously valued latent
variable process to be evaluated in a similar manner to Markov models for discrete valued time series, see MacDonald and Zucchini (1997). In doing so, this avoids the use of numerical integration or simulation schemes.

Under the DNF approach, the pdf of the latent variable, $x$, is approximated by computing the probability of observing $x$ within a set of discrete intervals (a histogram) as opposed to the linear spline approach suggested by Kitagawa (1987). In discretising state-space, $N$ adjacent intervals in $x$ space are defined, bounded by $w^1 \ldots w^{N+1}$, and centered on the points $x^1 \ldots x^N$ where

$$x^i = \frac{w^i + w^{i+1}}{2}.$$  \hspace{1cm} (7)

In general terms, the probability of observing $x$ within the interval centered on $x^i$, i.e. $x \in (w^i, w^{i+1}]$ is given by

$$p(x \in (w^i, w^{i+1}]) = \int_{w^i}^{w^{i+1}} f(x) \, dx \approx p(x^i)$$ \hspace{1cm} (8)

where $f(x)$ is the continuous probability distribution of the of the unobserved state variable $x$. The series of $p(x^i)$, $i = 1, \ldots, N$ represent a discretised approximation to the continuous distribution $f(x)$. Based on this discrete approximation, the DNF captures the evolution of the state variable through time given definitions of a time-invariant set of transition probabilities and a set of conditional likelihoods.

**Transitional probabilities**

Given that the state space is defined over $N$ adjacent intervals it is possible to compute an $N \times N$ matrix of time-invariant transition probabilities, $\hat{q}$. The elements of this matrix, $\hat{q}^{ij}$ $\forall i, j = 1, \ldots, N$, represent the probability of $x$ migrating from the interval centred on $x^j$ at time $t - 1$, to the interval centred on $x^i$ at time $t$ and is given by

$$\hat{q}^{ij} = \delta q(x^i_t | x^j_{t-1}, \theta)$$ \hspace{1cm} (9)
where \( \delta \) is the interval width. In the case where \( q(.) \) is a normal distribution,
\[
\hat{q}^{i,j} = \frac{\delta}{\sqrt{2\pi\sigma_v^2}} \exp\left( -\frac{(x^i - \alpha - \beta x^j)^2}{2\sigma_v^2} \right). \tag{10}
\]

**Conditional likelihoods** The likelihood of observing \( y_t \) conditional on \( x \) being within each discrete interval is found. The \( T \times N \) likelihood matrix containing elements, \( \hat{r}_t^i \ \forall i = 1, ..., N \), is defined by
\[
\hat{r}_t^i = r(y_t \mid x_t^i, \theta) \tag{11}
\]

In the standard SV model of equation 2, \( \hat{r}_t^i \) is given by
\[
\hat{r}_t^i = \frac{1}{\sqrt{2\pi \exp(x^i)}} \exp\left( -\frac{y_t^2}{2 \exp(x^i)} \right) \tag{12}
\]

Based on this set of conditional likelihoods and the time-invariant matrix of transition probabilities, the DNF proceeds with the following steps.

**Prediction Step**

The predicted joint probability of observing \( x \in (w^i, w^{i+1}] \) at time \( t \) and \( x \in (w^j, w^{j+1}] \) at time \( t-1 \) is given by:
\[
P_t^i = p(x_t^i \mid y_{t-1}, \theta) \tag{13}
\]
\[
= \sum_{j=1}^{N} q(x_t^i \mid x_{t-1}^j, \theta) p(x_{t-1}^j \mid y_{t-1}, \theta) \]
\[
= \sum_{j=1}^{N} \hat{q}^{i,j} \cdot U_{t-1}^j.
\]

**Update Step**

The updated probability of observing \( x \in (w^i, w^{i+1}] \) at time \( t \), is defined as
\[
U_t^i = p(x_t^i \mid y_t, \theta) \tag{14}
\]
\[
= \frac{r(y_t \mid x_t^i, y_{t-1}, \theta) p(x_t^i \mid y_{t-1}, \theta)}{p(y_t \mid y_{t-1}, \theta)}
\]
\[
= \frac{\hat{r}_t^i \cdot P_t^i}{p(y_t \mid y_{t-1}, \theta)}
\]
Likelihood

The denominator of equation (14) is the likelihood of observing $y_t$, given by

$$p(y_t | y_{t-1}, \theta) = \sum_{i=1}^{N} r(y_t | x^i_t, y_{t-1}, \theta)p(x^i_t | y_{t-1}, \theta)$$

$$= \sum_{i=1}^{N} \tilde{r}^i_t \cdot P^i_t$$

The advantage of the DNF procedure in this context is that the numerical integration required to evaluate equations 4 through 6 have been replaced by matrix operations used in a discrete valued Markov setting.

The log-likelihood used to generate ML estimates of $\theta$ are obtained directly from equation 15 and is given by

$$\ln L = \sum_{t=1}^{T} \ln[p(y_t | y_{t-1}, \theta)].$$

For the DNF to be initialised, the prediction of the state probabilities at time $t = 0$ need to be selected. The state probabilities are initialised by discretising the unconditional distribution of the state variable such that

$$P^i_1 = \int_{w^i}^{w^i+1} f(x | \theta) \, dx$$

where

$$f(x | \theta) \sim N\left(\frac{\alpha}{(1 - \beta)}, \frac{\sigma_w^2}{(1 - \beta^2)}\right).$$

The manner in which this DNF framework can be augmented to incorporate size effects will now be discussed.

3.2 Estimation of SV models with a size effect using DNF

To capture a size effect in SV dynamics it is necessary to adjust both the nonlinear filtering framework of equation 4 through 6 and the the associated estimation
procedure. In terms of the non-linear filtering equations, only the prediction equation, equation 4 must be adjusted to reflect the size effect,

\[
f(x_t | y_{t-1}, \theta) = \int_{-\infty}^{\infty} [I q(x_t | x_{t-1}, \theta_2) + (I-1) q(x_t | x_{t-1}, \theta_1)] f(x_{t-1} | y_{t-1}, \theta_1, \theta_2) dx_{t-1}
\]

(19)

where \( I = 1 \) if \( x_{t-1} > \tau \), with SV dynamics being governed by \( \theta_2 = (\alpha_2, \beta_2, \sigma_{w,2}) \) otherwise \( I = 0 \) if \( x_{t-1} < \tau \) results in dynamics being governed by \( \theta_1 = (\alpha_1, \beta_1, \sigma_{w,1}) \). This specification is consistent with equation 3 in that the dynamics governing the evolution of volatility at any point in time is dependent on the current level of volatility.

To estimate an SV model that captures such a size effect, three adjustments within the DNF estimation procedure must be made. First it is necessary to choose \( \tau \) so as the state-space of \( x \) may be partitioned into two adjoining regions. Within each region, discrete intervals must be chosen so as to discretise state-space. Finally, it is necessary to adjust the transition probability matrix, \( \hat{q}^{i,j} \) to reflect the distinct volatility dynamics of each region. Elements of \( \hat{q}^{i,j} \) will be computed as in equation 10 using the parameters from equation 3 which are associated with the region containing \( x_{t-1} \). Doing so implies that the probability mass of volatility within each region will be integrated forward using the respective sets of transition probabilities.

**Region Choice**

Whilst the state space of \( x \) is theoretically infinite, as with the standard SV case it must be discretised into a finite number of intervals. In the standard SV model these intervals are chosen to span \( \frac{\alpha}{(1-\beta)} \pm 6\frac{\sigma_w}{\sqrt{1-\beta^2}} \) where \( \alpha, \beta, \) and \( \sigma_w \) are given in equation 2. As the size effect, requires the use of two state equations, it is not immediately obvious how to span the state space of \( x \). However, it is reasonable to assume that volatility would still lie within the same region, even after considering the size effect, thus it is proposed that the state-space be defined
such that it spans the same region as that implied by the standard SV model. A first step is to find the ML estimates of the parameters of the SV model and compute

\[
\begin{align*}
\max(x) &= \frac{\hat{\alpha}}{(1-\hat{\beta})} + 6\frac{\hat{\sigma}_w}{\sqrt{(1-\hat{\beta}^2)}} \\
\min(x) &= \frac{\hat{\alpha}}{(1-\hat{\beta})} - 6\frac{\hat{\sigma}_w}{\sqrt{(1-\hat{\beta}^2)}}
\end{align*}
\] (20)

The region defined between \(\min(x)\) and \(\max(x)\) is believed to span the relevant state-space.

To split the state space of \(x_t\) into two regions, the threshold point, \(\tau\) is defined under a restriction that \(\tau \in \left[\frac{\max(x)+11\min(x)}{12}, \frac{11\max(x)+\min(x)}{12}\right]\). This ensures that there is a non-trivial distance between \(\tau\) and the edges of the discretisation, \(\min(x)\) and \(\max(x)\).

**Interval Choice**

It is now necessary to define the discretisation within each region, \([\min(x), \tau]\) and \([\tau, \max(x)]\). Define number of intervals in the upper and lower regions as

\[
N_U = \text{round}\left(\frac{N}{\max(x) - \min(x)}\right), \quad N_L = N - N_U
\] (21)

with the interval widths in each region \(\delta_U = (\max(x) - \tau)/N_U\) and \(\delta_L = (\tau - \min(x))/N_L\) respectively. Define a set of interval edgepoints to discretise state-space. These edgepoints are defined by \(w^1, \ldots, w^{N+1} = \min(x), \min(x)+\delta_L, \ldots, \min(x)+(N_L-1)\delta_L, \tau, \tau+\delta_U, \ldots, \tau+(N_U-1)\delta_U, \max(x)\). In a similar fashion to the standard SV model, the centre of each interval, \(x^1, \ldots, x^N\), is defined as the mid-point as in equation 7.

**Transit Probs**

To condition the transition dynamics on the level of volatility it is necessary to compute the matrix of transition probabilities, \(\hat{q}^{i,j}\), such that it reflects the
parameter values of the region to which $x^j$ belongs. Based on the volatility dynamics given in equation 3,

$$
\hat{q}^{i,j} = \frac{\delta_i}{\sqrt{2\pi \sigma_{w,s}^2}} \exp \left( \frac{-(x^i - \alpha_s - \beta_s x^j)^2}{2 \sigma_{w,s}^2} \right)
$$

(22)

where $S = 2$, if $x^j > \tau$, otherwise $S = 1$ and $\delta_i = \delta_U$ if $x^i > \tau$, otherwise $\delta_i = \delta_L$.

Given the definition of the two regions of state-space and the associated intervals, it is necessary to initialise the distribution of volatility, $P_i$. The simplest approach to initiating the distribution of volatility is to use estimated parameters from a standard SV model and initialise the distribution given equation 17 as if it were a standard SV model. While there are two sets of dynamics governing volatility in this threshold model, this initialisation will have no discernible impact on the performance of the filtering algorithm. Upon specifying $P_i$, the filtering algorithm proceeds as outlined earlier, recursing through equations 13 to 15 where the transition probabilities are now computed given 22 as opposed to 10.

### 3.3 Testing the significance of the size effect

Under the null hypothesis of $\theta_1 = \theta_2$ the SV model with size effects collapses to the standard SV model for any value of $\tau$. Since $\tau$ is unidentified under this null, a standard likelihood ratio (LR) test will not follow a standard distribution. Therefore to obtain accurate inference regarding the adequacy of the size effect, it is proposed that the non-standard distribution of the LR statistic can be determined by the use of a bootstrap procedure.

This procedure can be summarised in the following steps:

1. Estimate the parameter vector, $\hat{\theta}_{SV}$, of the standard SV model on actual data and store the log-likelihood, $L_{SV}$.

2. Estimate the parameter vector, $\hat{\theta}_{SIZE}$, of the SV model with size effects on actual data and store the log-likelihood, $L_{SIZE}$.
3. Find the likelihood ratio statistic $LR = 2 \times (L_{SIZE} - L_{SV})$.

4. Set $i = 1$.

5. Simulate a return series of length $T$, from the standard SV process using the parameter vector $\hat{\theta}_{SV}$.

6. Estimate the parameter vector $\hat{\theta}_{SV,i}$ of the standard SV model on the simulated data and store the log likelihood $L_{SV,i}$.

7. Estimate the parameter vector $\hat{\theta}_{SIZE,i}$ of the SV model with size effects on the simulated data and store the log likelihood $L_{SIZE,i}$.

8. Find the likelihood ratio statistic $LR_i = 2 \times (L_{SIZE,i} - L_{SV,i})$

9. Set $i = i + 1$ and repeat steps 5 – 8 until $i = N_{sim}$.

10. The empirical $p$ – value is then found as $1/N_{sim} \sum_{i=1}^{N_{sim}} I_i$ where $I_i = 1$ if $LR_i > LR$ and 0 otherwise.

4 Empirical results

Two datasets are considered for the empirical application of the SV model with size effects. Equity returns consisting of 2000 daily return observations from the S&P 500 index spanning 5 September 1996 to 16 August 2004 are utilised. Currency returns in the form of 2000 daily YEN/USD observations spanning 29 November 1996 to 30 July 2004 are also considered. Both datasets have been standardised to zero mean and unit variance.

Parameter estimates for both the standard SV and SV with size effect, along with tests of significance are outlined in table 1. As a benchmark, the results for the standard SV model are first addressed. These results reflect the commonly observed feature of relatively high persistence in conditional volatility. In comparison to these results, allowing for a size effect reveals a number of interesting features.
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Table 1: Estimation results for standard SV models and SV model with size effects (SIZE) for both the SP4500 and YEN/USD datasets. p-values for the LR statistic are generated from the bootstrap approach outlined above with Nsim = 500.

The most obvious feature evident once the size effect has been introduced is the differences between $\hat{\theta}_1$ and $\hat{\theta}_2$ for both series. When volatility is relatively low ($< \tau$) it is more persistent the standard SV case. Conversely, when volatility is quite high ($> \tau$) the persistence in volatility is much lower than the persistence found in either the low volatility region or the standard SV case. It is also evident that the variability of volatility is quite low (high) in the low (high) volatility regions.

In both cases the likelihood ratio tests indicate that the size effect is clearly an important feature of the respective datasets. This implies that the conditional volatility of these series are not linear processes in that the dynamics of volatility is dependent upon the level of current volatility.
5 Conclusion

Much research attention has been paid to the dynamics governing the evolution of financial asset return volatility. Apart from the common pattern of highly persistent volatility, two further features of volatility dynamics are of interest. These are the sign (level of volatility related to sign of past returns) and size (volatility dynamics related to current level of volatility) respectively. The asymmetric sign effect has been dealt with by numerous authors within both the GARCH and SV contexts. The size effect on the other hand has attracted much less attention with it not being considered in the context of an SV model.

The central contribution of this paper has been to propose a non-linear filtering based approach to the estimation of an SV process with size effects. A simple hypothesis testing procedure was also suggested to determine the significance of the size effect. While such a model has been considered here, the proposed DNF estimation procedure could be applied to a wider range of latent variable models where it is believed the dynamics of the latent variable is related to its level. This has been achieved by partitioning the possible state-space into adjoining regions and utilising region specific transition probabilities within the prediction step within DNF algorithm.

Empirical application of the SV model with size effect shows that it is certainly an important feature of the two series considered here. Given the equity and currency returns considered, volatility dynamics appear to be dependent upon the current level of volatility. In both instances, the persistence of volatility falls and the volatility of volatility rises as the current level of volatility rises, suggesting that volatility dynamics are not linear.

References


