Do agents maximise? Risk taking on first and second serves in tennis.*

Jeffrey Ely¹, Romain Gauriot², and Lionel Page³

¹Northwestern University

²University of Sydney

³Queensland University of Technology

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Abstract

We investigate whether expert players with high incentives are able to optimally determine their degree of risk taking in contest. We use a large dataset on tennis matches and look at players risk taking on first and second serves. We isolate a specific situation, let serves, where second serves and first serves occur in a way which is as good as random. This creates the setting of a quasi-experiment which we can use to study players' serving strategies on first and second serves in comparable serving situations. We find that players, both men and women, are able to adopt serving strategies which meet the requirements of optimality arising from simple assumptions about risk-return trade-offs in serves.

It's an insidious disease of backing off the second serve after they miss the first serve. *Bill Tym, Vanderbilt Coach*

Nine of the top 20 men as of the Aug. 2 rankings would be better off statistically or virtually unaffected by using their first-serve technique on the second serve. *New York Times, Aug 29, 2010*

1 Introduction

Game theory has been one of the most successful theoretical areas in economics. It is now commonly taught in economics curricula and has spread beyond economics to a wide range of other disciplines such as biology, sociology, political science. In spite of this success, it still faces questions about its relevance for strategic interactions in the real world. The abstract and simplifying approach from game theory often faces the question “What is its empirical content?”. A very large literature has investigated predictions from game theory in laboratory experiments. The overwhelming conclusion from these studies is that standard game theoretic assumptions often fail to predict how individuals behave in

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strategic interactions, and a wide range of cognitive limitations and psychological biases have been observed (Camerer 2003). In response to this evidence, other studies have looked for situations where game theory is more likely to be able to describe behaviour: strategic interactions with players who are experts and have high incentives. The nature of professional sports fits these conditions and thus provides a wealth of data on strategic behaviour. They have become an ideal testbed for game theoretic predictions. Walker and Wooders (2001)’s seminal paper found that tennis players approximate minimax strategies when choosing the direction of their serves in tennis. This result has been followed by similar ones in other sports and situations suggesting that professional players are actually reasonably good at approximating optimal/equilibrium strategies (Palacios-Huerta 2003, Abramitzky, Einav, Kolkowitz, and Mill 2012, Gauriot and Page 2014, among others). The conclusion of these studies is that, for experts and well incentivised agents, game theory predictions are often supported by the empirical evidence.

The present study contributes to this literature by studying whether professional players are able to optimally allocate their risk taking between two consecutive strategic decisions. Several papers have touched on this subject before using tennis as a setting (George 1973, Gillman 1985). The closest related paper is by Klaassen and Magnus (2009). They use data from four years at Wimbledon to estimate a structural model of serving strategy and assess the hypothesis of optimality. They find evidence that players’ behaviour approximate the optimal allocation of risk taking between serves. Our study complements and strengthens their result by eliminating concerns about a potential caveat with raw data on tennis serve: it can be characterised by a selection problem. First serves are always observed while second serves are only observed when the first serve failed. This means that second serves may be more likely to be observed when serving is harder than usual either for natural reasons (e.g wind conditions), fitness (e.g. tiredness late in the match) or strategic reasons (e.g. opponent having learned how to return the player’s serve).

To cleanly compare first and second serves one ideally wants to observe some random events which determines in a given situation whether a serve is going to be a first or a second serve. We argue that such a situation occurs when the ball hits the tape (top of the net) on the first serve. The impact with the net gives the ball an unpredictable trajectory leading the ball to be either in or out. It introduces the required randomness as a first serve follows a ball let which lands in the court and a second serve follows a ball which lands outside the court. We use a large and precise ball-tracking data set on professional tennis player matches to isolate let serves. This quasi-experiment set-up allows us to compare the risk taking between first and second serves across situations which are going to be comparable in terms of the relevant strategic dimensions.

The focus of this study on the allocation of risk-taking complements the existing studies on professional sport players and game theoretic predictions. Previous studies have shown that players are able to play the mixed strategy (Walker and Wooders 2001) or to approximate optimal strategies like using an option at the right time (Abramitzky, Einav, Kolkowitz, and Mill 2012). In addition to these questions, sports competitors often face a trade-off between safe options with low gains and risky options with potentially high gains (and higher probability of failure). Choosing a risk-taking strategy is in itself non-trivial. Therefore it is unclear whether we should expect players to find the optimal level of risk-taking as a function of the situation.

In this study, we consider the decision of tennis players to take risk on their serves. When serving, players can opt for risky serves which are more likely to fail but are harder

to return if successful or more conservative serves which are less likely to fail but are also easier to return. Players' choice depend on whether they play a first or second serve. When serving, players have two opportunities to land a correct serve. If their first serve does not land within the correct boundaries on the court ("serving box"), they are given a second chance with a second serve. This structure is interesting beyond tennis. It corresponds to a common situation where players need only to succeed once in two attempts. The possibility of a second attempt allows players to take more risk on their first attempt. One can think of several illustrative examples in different contexts. Faced with the prospect of two job interviews for similar jobs, an agent can choose to be more demanding in his first interview knowing he/she still have a chance in a second interview. In a primary race, a politician needing only one victory out of two states to secure the nomination may decide to invest less effort/money in the first state knowing that there is a second opportunity to secure the nomination with the second state.

We find that tennis players respect a set of four conditions imposed by optimal serving behaviour according to a simple model of the risk-return trade-off faced by players when serving. Unlike other studies on tennis players strategies we do not find significant gender differences with female players respecting the conditions of optimality like male players. We also do not find difference related to players' ability (ranking) or experience (age).

The remainder of the paper is as follows: Section 2 presents our theoretical framework; Section 3 describes the data; Section 4 presents the results and Section 5 concludes.

2 Conceptual framework

We use the model proposed by Klaassen and Magnus (2009). Let's consider the situation of a tennis player having to decide how much risk to take on in each of the two serves. We define x_i as the probability that serve i , ($i \in 1, 2$) goes in; $y(x_i)$ as the conditional probability that the player wins the point if serve i goes in; $w(x_i) = x_i \times y(x_i)$ as the probability to win the point on serve i for level of risk on the serve x_i . The players faces a risk-return trade-off between x_i (1-risk of failure of the serve) and y_i (likelihood of successful serves to win the point). Similarly to Klaassen and Magnus, we assume two conditions:

Condition 1 (Monotonicity). *The function $y(x)$ is decreasing on $[0, 1]$, with $y(x) \in [0, 1], \forall x \in [0, 1]$.*

This condition reflects the intuitive idea that a higher conditional probability y can only be achieved at the cost of a greater risk in serve (speed and ball location). This greater amount of risk is associated with a greater probability of failure (lower x).

Condition 2 (Concavity). *The function $y(x)$ is twice differentiable on $[0, 1]$, with $y'(1) < 0$ and $w'(x) < 0, \forall x \in [0, 1]$.*

This condition implies that the probability to win the point as a function of the risk taken on a serve is inverse U-shaped. For low amounts of risk (high x), serves are too easy to return for the receiver. Increasing risk-taking increases then the winning probability for the server. However, at some point, the serves become too risky (low x) and even though they are very hard to return when inside the court, they are too often outside the court and do not allow the server to win the point. This condition implies that there is an intermediate level of risk x which is such that the probability to win the point w

is maximal. Under these two conditions, the following inequalities hold (Klaassen and Magnus 2009, Theorem 2)

Proposition 1 (risk tradeoff).

- (i) *First serves should be more risky than second serves:*
 $x_1 < x_2$.
- (ii) *First serves should be harder to return than second serves:*
 $y_1 > y_2$.
- (iii) *Using two first serves is a suboptimal strategy, leading to the inequality:*
 $w(x_1) < w(x_2)$.
- (iv) *Using two second serves is a suboptimal strategy, leading to the inequality:*
 $w(x_2)(1 + x_1 - x_2) - w(x_1) < 0$.

Using a data set of point-by-point data from Wimbledon matches, Klaassen and Magnus investigate these predictions, as well as the degree of optimality of serves, using a structural approach on the whole dataset. They find that players approximate the optimal strategy estimated by their model. Our approach complement theirs.

One issue when taking all observed first and second serves is that second serves may be more likely to be observed when serving conditions are more difficult due to a tougher opposition, weather condition or even due to the server's fitness. As serve conditions may vary across and within matches, the function $y(x)$ will vary too. Comparing observed first and second serves may therefore lead to comparing these serves across situations with different $y(x)$. Ideally, one would want to compare successful first serves with the counterfactual second serves: the second serves which would be observed if the first serves were unsuccessful. In order to achieve this, we focus on a quasi-experimental situation where the occurrence of a first serve or a second serve is as good as random: when the first serve is "let". As a consequence we consider second serves in situation where they were as likely to occur as first serves given the existing serving conditions. Our observed second serves can therefore be considered as counterfactual observations to the first serves and we can assume that the function $y(x)$ does not differ significantly between first and second serves in this quasi-experimental situation.

Figure 2.1 presents typical examples of let serves going either in or out and leading respectively to a new first serve or to a second serve. The figure represents actual serves observed in our dataset.

Figure 2.1: Examples of trajectories of let serves observed in our dataset (a) serve going in from Zheng, semi-final of Wimbledon 2008 against S. Williams, (b) serve going out from Roddick, 1st round of the Australian Open 2006 against Lammer.

3 Data

Our dataset corresponds to the official Hawk Eye ball-tracking data. The Hawk Eye system is composed of several cameras installed around the courts and track the trajectory of the ball at all times. In tennis when players “challenge” the decision of the umpire the Hawk eyed system is used in order to determine whether the ball landed inside the court or not¹. We have almost all the matches played at the international professional level between 2005 and 2009, where this technology was used. Most of the matches are either from Grand Slam and ATP (Association of Tennis Players) tournaments, or lower level ITF (International Tennis Federation) tournaments. In addition, some matches are from diverse cups like the Davis Cup or the Olympic games and some exhibition tournaments.

We have 3,188 matches which correspond to 691,360 serves. Part of the data is entered by hand which introduces an increased probability of some inconsistencies due to data entry errors. In order to ensure the highest quality possible we disregard games with discrepancies in the evolution of the score. We are left with 478,027 serves (that is 69% of the data). From this dataset we extract the subset of situations when the first serve is a “let” (i.e. it hits the top of the net). On a first serve, if the ball hits the net and goes in, the server has another first serve and if the ball goes out, the server plays a second serve. That gives us a random generation of first and second serves.

Here the “lets” are defined as balls hitting the net and crossing it, or ball hitting the net and bouncing up off the net backward. This is not an exhaustive definition of the “let”.² We have identified 7,605 lets, 1,283 land in (and are thus followed by a first serve) and 6,322 are landing out (and are followed by a second serve). Table 1 gives descriptive statistics for these specific points as well as the overall sample.

In addition to the Hawk Eye data, for a large subsample of the matches we collected the ATP and WTA player’s ranking at the time of the matches, as well as the bookmaker odds prior to the match. We were able to get this information for 94% of male players’

¹More information can be found on the Hawk Eyed official website: <http://www.hawkeyeinnovations.co.uk/sports/tennis>.

²We focused on selection to the situations where the ball hits the top of the “tape” at the top of the net. We for instance did not consider balls hitting the top of the net and dropping right on the ground on the wrong side of the net as these could be located low relatively to the top of the net.

		Number of serves			Lets on 1 st serve		
Event Features		Female	Male	All	Female	Male	All
Surface	Carpet	5,990	28,855	34,845	130	620	750
	Clay	15,360	58,796	74,156	195	751	946
	Grass	14,274	39,012	53,286	151	487	638
	Hard	122,927	192,813	315,740	1,947	3,324	5,271
Best of	3	158,551	196,440	354,991	2,423	3,046	5,469
	5	-	123,036	123,036	-	2,136	2,136
Event	Davis Cup (<i>Fed Cup</i>)	848	4,698	5,546	15	105	120
	Grand Slam	58,591	109,164	167,755	842	1,830	2,672
	Olympics	3,051	2,469	5,520	75	48	123
	ATP (<i>Premier</i>)	90,511	1,478	91,989	1,410	34	1,444
	International	-	71,247	71,247	-	1,133	1,133
	Master	-	123,917	123,917	-	1,920	1,920
	Hopman Cup	5,550	6,503	12,053	81	112	193
Total		158,551	319,476	478,027	2,423	5,182	7,605

Table 1: Descriptives statistics on the serves in the overall dataset and lets on first serve. Name in italic for womens' tournaments.

matches and 68% of the female players' matches.³

A key element of our identification strategy is that let situations create the quasi-experimental set-up where the serving conditions, summarised by the function $y(x)$, are on average the same for first and second serves. In order to test the validity of this assumption, we compare in Table 2 the difference between the players putting a let in and those putting a let out.

First, we test for difference in players' ranking. The ATP/WT A ranking has a pyramidal structure; *i.e* the difference in ability between players ranked 1 and 2 is greater than between player ranked 50 and 51. Thus we follow Klaassen and Magnus (2009) and use the measure of ranking $R = 8 - \log_2(Rank_{ATP/WT A})$ (henceforth KM ranking). We observe that both for males and females there is no significant differences in the rankings of the servers (Line 1) and of the receiver (Line 2). To complement players' rankings, we also use the betting odds, prior to the match, for the player winning the match. We reverse these betting odds to estimate the player's winning probability as estimated ex ante by the bookmakers. Here again we find no significant difference between servers putting the ball in or out after a let (Line 3). Finally, we use the information on the match itself. We find that servers who put the ball in are not more likely to be ahead (or behind) in a game than their opponent than servers who put the ball out (Line 4). All these results support our identification hypothesis that let situations create the set-up of a quasi-experiment. First and second serves following a let can therefore be considered as being generated from situations with similar serving conditions.⁴

³We use the data available at <http://www.tennis-data.co.uk/>, and we use the odds from Bet365 available in this dataset. This dataset does not cover the years 2005 and 2006 for female players, which explain why we were not able to get this information for as many male matches than female matches.

⁴All the above comparisons are meaningful as comparison within a given match. We also investigated differences in probability to put the serve in/out after a let across matches with different characteristics.

	Women	Men
Server's KM ranking	0.029	-0.109
p	0.801	0.176
N	1,869	4,849
Receiver's KM ranking	0.079	0.0436
p	0.470	0.594
N	1,867	4,849
Ex ante probability of winning	0.012	-0.001
p	0.718	0.604
N	1,835	4,812
Server ahead in game	0.009	-0.009
p	0.589	0.387
N	2,423	5,182

Table 2: Difference in ability between server putting a let in and the ones putting a let out on first serve.

4 Results

4.1 Overall in the sample

First, we test the predictions of Proposition 1 on the whole dataset. Then we present the results of the same tests on our quasi-experimental sample. Table 3 presents the tests of the four predictions from Proposition 1.

Result 1 (Optimality in the main sample). *The conditions for optimality are respected in the overall sample of all the serves.*

Table 3 also presents the results split by gender. Noticeably both men and women meet all the conditions for optimality. There is therefore no indication in the overall data that players exhibit a systematic suboptimal serving behaviour in the form of a too risky or too conservative second serve.

Our identification strategy allows us to go beyond these descriptive statistics and focus on situations where serve conditions are similar when first and second serves are observed. It is only in such conditions that, formally, the conditions from Proposition 1 have to be respected. Table 4 presents the results comparing first serve and second serve outcomes in quasi-experimental sample of serves following a let situation. These results confirm the pattern from the overall data: the four inequalities of Proposition 1 are respected. Furthermore, these inequalities are respected both for men and women. We do not observe any indication of gender difference.

Result 2 (Optimality in the quasi-experimental sample). *The conditions for optimality are respected in the quasi-experimental sample of serves following a let first serve.*

We found that serves are more likely to be in after a let for men matches and for matches in best of 5 sets. Potentially, this could create some spurious differences in our sample when aggregating across matches, for this reason, we split all our sample by gender for the results. We also checked all our results stayed identical excluding best of 5 matches.

	Overall		Men		Women	
	1 st serve	2 nd serve	1 st serve	2 nd serve	1 st serve	2 nd serve
x_i	0.663	0.914	0.662	0.931	0.664	0.879
$s.e$	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	0.002
Inequality (i)	✓		✓		✓	
y_i	0.635	0.541	0.659	0.555	0.587	0.512
$s.e$	0.001	0.002	0.001	0.002	0.002	0.003
Inequality (ii)	✓		✓		✓	
$w(x_i)$	0.421	0.495	0.436	0.517	0.390	0.450
$s.e$	< 0.001	0.001	0.001	0.002	0.001	0.002
Inequality (iii)	✓		✓		✓	
$w_2(1 + x_1 - x_2) - w_1$	-0.051		-0.059		-0.037	
$s.e$	0.001		0.001		0.002	
Inequality (iv)	✓		✓		✓	
N	349,277	128,750	233,108	86,368	116,169	42,382

Table 3: Results in the overall data. The value y_i is estimated using only serves which are landing in. Hence, $N * x_i$ serves are used to estimate y_i . Inequality (iv)'s standard error are computed using 1,000 bootstrap samples.

1. *First serves are more risky than second serves:*
 $x_1 = 0.682 < x_2 = 0.912$
2. *First serves are harder to return than second serve:*
 $y_1 = 0.629 > y_2 = 0.537$
3. *Using two first serves is a suboptimal strategy, leading to the inequality:*
 $w(x_1) = 0.429 < w(x_2) = 0.490$.
4. *Using two second serves is a suboptimal strategy, leading to the inequality:*
 $w(x_2)(1 + x_1 - x_2) - w(x_1) = -0.052 < 0$

One way to give light to these results is to show what would happen if players were to deviate from these observed frequencies to follow the advice from the quote at the start of this article and were to adopt two first serves rather than one first and one (conservative) second serve. Let's consider a male player. Using the numbers from Table 4, the probability of winning the point when serving a first and second serve is $P(win_{12}) = w(x_1) + (1 - x_1)w(x_2) = 61.6\%$. If instead the player opted to use two first serves, his probability to win the point would only be $P(win_{11}) = w(x_1) + (1 - x_1)w(x_1) = 59.9\%$. Similarly, if he was to use two second serves, his probability to win the point would be $P(win_{22}) = w(x_2) + (1 - x_2)w(x_2) = 54.4\%$.

We investigate these results further by looking at two representative players and a median split of our sample. Table: 5 shows the statistics for the male and female servers most present in our dataset: Rodger Federer and Jelena Jankovic. For Federer (Jankovic) we observe 19,792 (7,476) serves and 290 (117) lets on first serve from 231 (102) matches.

	Overall		Men		Women	
	1 st serve	2 nd serve	1 st serve	2 nd serve	1 st serve	2 nd serve
x_i	0.682	0.912	0.681	0.931	0.684	0.869
$s.e$	0.013	0.004	0.016	0.004	0.021	0.008
Inequality (i)	✓		✓		✓	
y_i	0.629	0.537	0.667	0.546	0.563	0.515
$s.e$	0.016	0.007	0.020	0.008	0.028	0.012
Inequality (ii)	✓		✓		✓	
$w(x_i)$	0.429	0.490	0.454	0.509	0.385	0.448
$s.e$	0.014	0.006	0.018	0.008	0.022	0.011
Inequality (iii)	✓		✓		✓	
$w_2(1 + x_1 - x_2) - w_1$	-0.052		-0.073		-0.020	
$s.e$	0.013		0.015		0.020	
Inequality (iv)	✓		✓		✓	
N	1,283	6,322	808	4,374	475	1,948

Table 4: Results in the quasi-experimental sample. $N * x_i$ serves are used to estimate y_i . Inequality (iv)'s standard error are computed using 1,000 bootstrap samples.

For the period of our dataset Federer was either number 1 or 2 in the ATP ranking. Jankovic was rank from 1 to 11 at the WTA ranking. Table 5 also compares players who are higher and below the median in ranking. The median ranking for male is 21 and the median ranking for women is 17.

We observe that the inequalities are respected in almost all cases. However, 2 out of the 24 inequalities from Table 5 go in the wrong direction. These are inequality (iii) for Jankovic and inequality (iv) for high ranked female players. To assess whether these violations are significant we bootstrapped the values from these inequalities. We found these two inequalities not be significant. The overall picture is therefore that the conditions from Proposition 1 are almost all the time respected not only overall but also for both genders and for high and low ranked players.⁵

⁵Note that given the small number of let serves per player we are not able to do an analysis at the level of individual players. We can only do so for players with the largest number of observations, like Federer and Jankovic.

	Male						Female					
	Rodger Federer		Higher ranked		Lower ranked		Jelena Jankovic		Higher ranked		Lower ranked	
	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd
x_i	0.806	0.949	0.698	0.935	0.660	0.925	0.786	0.787	0.697	0.851	0.665	0.868
$s.e$	0.067	0.014	0.024	0.005	0.024	0.006	0.079	0.044	0.033	0.013	0.035	0.013
Inequality (<i>i</i>)	✓		✓		✓		✓		✓		✓	
y_i	0.655	0.569	0.675	0.555	0.658	0.531	0.591	0.543	0.564	0.562	0.552	0.488
$s.e$	0.090	0.032	0.030	0.011	0.030	0.012	0.107	0.060	0.042	0.020	0.045	0.020
Inequality (<i>ii</i>)	✓		✓		✓		✓		✓		✓	
$w(x_i)$	0.528	0.539	0.471	0.519	0.434	0.491	0.464	0.427	0.393	0.478	0.367	0.424
$s.e$	0.084	0.031	0.027	0.010	0.025	0.011	0.096	0.053	0.035	0.013	0.035	0.013
Inequality (<i>iii</i>)	✓		✓		✓		✗		✓		✓	
$w_2(1 - x_2 + x_1) - w_1$	-0.066		-0.075		-0.073		-0.038		0.011		-0.029	
$s.e$	0.080		0.022		0.021		0.100		0.034		0.033	
Inequality (<i>iv</i>)	✓		✓		✓		✓		✗		✓	
N	36	254	357	2,074	385	2,033	28	89	201	753	188	727

Table 5: Results for Rodger Federer (column 2) and Jelena Jankovic (column 5), the male and female players the most present in our dataset, and players who are ranked above (column 3 and 6) and below the medium rank (column 4 and 7) in the quasi-experimental sample. The medium rank is 17 for male and 21 for female players. $N * x_i$ serves are used to estimate y_i . Inequality (*iv*)'s standard error are computed using 1,000 bootstrap samples.

5 Conclusion

In this study we look at the ability of expert agents (tennis players) with high incentives to adopt an optimal level of risk in a well defined strategic situation (serving) within a contest (tennis match). We find that professional tennis players are able to meet the requirement of the optimal strategy when deciding of the risk to take on their first and second serves. This result is observed both overall and when splitting the sample by gender and ranking. This result complements those from Klaassen and Magnus (2009) who used a structural econometric model to investigate the same question.

Even though game theory predictions have often failed to be supported by laboratory experiments, our results confirm previous results that the same predictions can be relevant to study the behaviour of expert agents with high incentives in the real world. One criticism often raised against game theory is that agents are unlikely to find the solution of complex strategic games. Certainly, we do not argue that tennis players find the optimal strategies consciously. However, with enough training, players may have learned the strategies which work and they may have acquired intuitions for the game which are well enough calibrated to allow them to generate strategies which approximate the optimal strategies in the game. Furthermore, as we look at a competition between top agents, the pool of players we observe has undergone a long selection process. Compared to the large pool of players who want to become top players, only a few make it. Those who are more able to approximate optimal strategies are more likely to have survived and be present in our dataset of professional players.

One may therefore stress that the situation we are looking at may be an ideal situation for the predictions of game theory: not only are our players experts and well incentivised, there have also faced a tough selection pressure on their ability to adopt the best strategies. One may conjecture that the more a situation will feature these three characteristics (expertise, incentives, selection pressure), the more game theory predictions are likely to be relevant to understand behaviour.

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