Does success breed success? A quasi-experiment on strategic momentum in dynamic contests.

Romain Gauriot*

Lionel Page

School of Economics and Finance
Queensland University of Technology

November 4, 2014

*For detailed and constructive comments we gratefully thank Changxia Ke, Kai Konrad, Peter Siminski, John Wooders and Oleg Yerokhin. We also benefited from comments from seminar participants at the Econometrics Society Australasian Meeting (Sydney), Australia New-Zealand Workshop in Experimental Economics (Brisbane), Economic Science Association (Auckland), French Economic Association Annual Meeting (Lyon), Maastricht Behavioural and Experimental Economics Symposium, French Experimental Economics Association Conference (Besancon), University of New South Wales, University of Wollongong, Ludwig Maximilien University, Innsbruck University, Sun Yat Sen University and Tsinghua University. Lionel Page thanks the Australian Research Council and the Queensland Department of Industry for funding.

Abstract

We investigate whether professional agents react to changes of incentives during dynamic contests as predicted by contest theory. Using a large data set of point by point ball tracking data from tennis matches over the period 2005-2009, we exploit the randomised variation in point results that occurs when balls bounce very close from the court's line to estimate the causal effect of winning a point on the chance to win the next point. In line with predictions from contest theory, we find evidence of a substantial momentum effect for male players. We do not find any significant effect for female players, suggesting the possible existence of gender differences in how agents react to incentives in contests.

Tournaments are pervasive in society and economic organisations such as promotion tournaments, patent races, rent seeking, litigation, and political contests (Konrad 2009, for a review of the literature). Moreover, tournaments have attracted the interest of economists for their possible use as incentive schemes to optimise the effort level of agents when effort is not fully observable Lazear and Rosen (1981). In recent years, the empirical research on contests has flourished. While Prendergast (1999) was stressing that there was a lack of empirical evidence about the effect of incentives on agents' behaviour in tournaments, economists have learned a lot since from applied work using real contests, in particular sporting contests (Taylor and Trogdon 2002, Chiappori, Levitt, and Groseclose 2002, Palacios-Huerta 2003, Walker and Wooders 2001, Romer 2006, Hsu, Huang, and Tang 2007, Pope and Schweitzer 2011, Genakos and Pagliero 2012) and laboratory experiments (for a survey of the large literature, see: Dechenaux, Kovenock, and Sheremeta forthcoming). However, as pointed by Carpenter, Matthews, and Schirm (2010), there is still much economists do not know about incentives in tournaments.

A particular challenge facing economists is to understand how agents react in dynamic contests when the agents' strategies have to take into account past and future actions of their opponents. Most real life tournaments take place over time and are de facto dynamic contests. Theoretical research has looked at the substructure of tournaments when they can be decomposed into smaller contests (Konrad 2009). The introduction of a time dimension makes optimal strategies typically more computationally demanding than one period contests (Klumpp and Polborn 2006, Konrad and Kovenock 2008).

Studying agents strategic behaviour in dynamic contests is relevant to understanding how contests and tournaments work in practice as incentive schemes. Unfortunately, the study of the causal effect of past performance in dynamic contest faces a difficult challenge: past performances are typically correlated with unobserved characteristics which influence future performances. This creates a potential endogeneity bias. This difficulty may explain the mixed evidence in the existing literature about whether agents’ strategic behaviour reacts to incentives in dynamic contests as would be predicted by theory (Ferrall and Smith Jr 1999, Tong and Leung 2002, Malueg and Yates 2010, Dechenaux, Kovenock, and Sheremeta forthcoming).

The present paper addresses the difficult problem of identifying the causal effect of winning on future performance using a quasi-experimental setting. In this setting we can investigate how agents adapt their behaviour in dynamic contest when their overall advantage relative to their opponent changes for reasons which are not correlated with differences in players characteristics. Specifically, we take a very large dataset on precise ball location during tennis matches between professional tennis players to extract a very small subset of points where the ball bounced within a few centimeters of the court lines. The rule of tennis implies that balls landing just out of the court lines lead to a loss of the point for the player who hit the ball. On the contrary, the game goes on if the ball landed just on the inside of the court lines, giving the player a positive probability to win the point. We use this discontinuity in the probability to win the point for ball landing very close to court lines to implement a fuzzy regression design (Imbens and Lemieux 2008) to investigate how professional players’ performance in tennis games change after winning or losing a point.

Tennis games are dynamic contests where points are sub-contests. Models of dynamic contests typically predict a “momentum effect” for the winner of a subcontest (also called “discouragement effect” for the losing player) (Konrad 2009). We show in this paper that this prediction extends to tennis games and that players winning points should most often increase their effort level in later points while losing players should decrease theirs, leading to a momentum effect. Noticeably, this prediction goes against the dominant view regularly expressed by professional players that every point should be considered in isolation with the player making the maximum effort to win each of them. For instance, Novak Djokovic, ranked number one player in the world at the time, declared in 2013 after winning a match at the US Open “I was wishing [...] to be able to stay committed to play every point,

to win every point, regardless of what's the score.”¹

In contrast with this idea and in line with predictions from contest theory, we find a significant momentum effect for male players: players are 5 to 10 percentage point more likely to win the next point after winning the present point. We use the richness of our dataset to test further predictions such that the momentum effect should be larger when the scoreline is symmetric and towards the end of a game. We find that these predictions are indeed observed for professional male players. Looking at a smaller dataset for females we do not find evidence of a momentum effect. This result adds to a substantial body of research has established the existence of gender differences in competition (Niederle and Vesterlund 2011, for a review). It suggests that part of these differences may arise from differences in how males and females react to incentives in dynamic contests.

The remainder of the paper is as follows: Section 1 places the contribution of this paper in the existing literature, Section 2 presents our conceptual framework, Section 3 presents our identification strategy and our dataset, Section 4 presents the results, and Section 5 concludes.

1 Related literature

While a wide range of designs exists for dynamic contests, existing models typically suggest that if agents react appropriately to incentives during a dynamic contest, a “strategic momentum” should appear. The perfect equilibria of races modelled by Harris and Vickers (1985) and Harris and Vickers (1987) lead to the leader of the race making greater effort than the follower. It is also a result found by Ferrall and Smith Jr (1999), Konrad and Kovenock (2008) and Malueg and Yates (2010) in the case of best-of-n contests and by Breitmöser, Tan, and Zizzo (2010) in the context of R&D races. On the other hand it has also been suggested that contestants follow a “hare-tortoise” heuristic where the “trailing contestant will exert more effort to catch up, whereas the leading contestant will slack off” (Tong and Leung 2002). Outside economics,

¹It is a principle which can also be found as “strategy” advice given by coaches: “Focus on that skill to the extent that you are not thinking about the score or who is winning”, “Play every point and return every shot” (Bryant 2011, p. 73, p. 90). Beyond tennis, it is common to hear this advice in other sports which are dynamic contests. The professional Canadian ice hockey player Guy Lafleur declared “Play every game as if it is your last one.”. The company Nike re-used this advice in a global advertising campaign at the start of the 2014 football world cup, inviting to “Play every game like it’s ‘The Last Game’”.

the notion of “psychological momentum” has been proposed as reflecting that winning may help enhance contestants’ confidence with the consequence that “success breeds success” (Dorsey-Palmateer and Smith 2004, Vallerand, Colavecchio, and Pelletier 1988).

The existing empirical literature presents conflicting results. A large body of literature in social psychology initiated by Gilovich, Vallone, and Tversky (1985) has argued that momentum does not exist in competitions and that the layman perception of momentum (so called “hot hand” in their initial study on basket ball players) is a cognitive illusion. This research has led to a substantial amount of empirical studies in social psychology which have provided mixed empirical results with a tendency to lean towards an absence of any momentum (Bar-Eli, Avugos, and Raab 2006). However, it has been shown that many of these studies use statistical tests with low power (Wardrop 1999).

On the contrary, studies in economics have been aimed at testing the results from game theoretical models suggesting the existence of momentum. Within the recent growth in laboratory experiments on contests, several studies have looked at this question in a controlled environment (Dechenaux, Kovenock, and Sheremeta forthcoming). The evidence here is also mixed. Tong and Leung (2002) and Fu, Ke, and Tan (2013) found results supporting a negative momentum effect such that trailing contestants expend more effort to catch up with leading contestants. On the contrary, Mago, Sheremeta, and Yates (2012) found experimental evidence consistent with the existence of a strategic momentum. The equilibrium strategies in dynamic contests are arguably complex and part of the mixed evidence may come from the unfamiliarity of participants with such strategic settings.

Looking at real contests in the field eliminates this concern. Several studies have looked at team competitions in sport contests. Ferrall and Smith (1999) estimated the parameters of a structural model of strategic behaviour in a best-of-7 contest using data from US championship series. They found an absence of momentum in these contests. Teams seem to play each game as well as they can. In an old study also using data from US Championship series, Simon (1971) actually found a negative momentum effect, labelled as a “back to the wall” effect. Using a large dataset on professional basket ball matches Berger and Pope (2011) found that teams who are just behind at half time are more likely to win the match, suggesting here again a negative

momentum effect.²

Ferrall and Smith pointed out that an individual competition like tennis may however provide more leverage to study strategic incentives than team competitions. The characteristics of tennis have indeed made this sport a workhorse setting to study players' behaviour in a strategic game in the field. It has been used to study whether players can play mixed Nash equilibrium strategies (Walker and Wooders 2001, Hsu, Huang, and Tang 2007), whether they optimise their chances of winning when making risk-return trade-offs (Klaassen and Magnus 2009, Abramitzky, Einav, Kolkowitz, and Mill 2012) and whether there are gender differences in performance in contests (Paserman 2007).

Two studies have indeed already used tennis data to investigate the changes of players' strategies in a dynamic contest. Looking at point by point data from the Wimbledon tennis tournament, Klaassen and Magnus (2001) found that the hypothesis that points are iid should be marginally rejected in tennis, with the possibility of a momentum effect in some cases between points. Malueg and Yates (2010) selected a sample of 268 tennis matches where players have identical betting odds found support for the existence of a momentum effect between sets of tennis matches. A player winning a first set by two games or less tends to win the second set more often than his opponent, even though arguably they are very close in ability.

The present study adds to this literature by looking at a quasi-experimental setting within a large sample of tennis games. Our strategy addresses a key difficulty when trying to estimate the effect of previous performance on later performance in the field. Past and future performances are naturally correlated due to unobservable characteristics which impact both of them. Even within a given dynamic contest, characteristics such as the fitness and ability of the players may not be constant over time (due to fatigue, injury, learning, or change in team composition for team contests). This hypothesis of constant underlying ability/skills of the contestants typically underlies field studies. The mixed evidence from the existing studies using field data may partly come from the fact that it is hard to eliminate the possibility that this hypothesis may not be respected. Our identification strategy eliminates such a confounding factor in order to cleanly identify the causal effect of previous

²In the context of political races, Klumpp and Polborn found some evidence of "momentum" in the US primaries and stress that the importance given to the first primary in the New-Hampshire could stem from the existence of such a momentum.

results on later performance (see Section 3).

2 Conceptual framework

Tennis is a dynamic contest composed of several subcontest levels: sets, game, points. Here we study the existence of a momentum between points, within a game. A tennis game has a structure which is very similar to best-of- n multiple battle contests. It imposes an additional constraint whereby the winner needs to have won two more battles than the other player. Figure 1 shows the structure of a tennis game.³ Players contest “points”. Starting from a state (s_A, s_B) in the game where s_A and s_B are the scores of player A and B respectively, players move to state $(s_A + 1, s_B)$ if A wins the point and to $(s_A, s_B + 1)$ if B wins. This contests structure is studied in detail by Walker, Wooders, and Amir (2011) in the case where players’ strategies are costless. When players’ strategies involve choosing a costly investment (e.g. effort) as assumed in the literature on dynamic contests, a tennis game is a hybrid design between a best-of- n contest which has a maximum of n sub-contests and a “tug-of-war” (Konrad 2009) at the end of the game where players can move (potentially endlessly) back and forth between the states $(3,2)$, $(2,2)$, $(2,3)$.

Given such a contest structure, we first define the notion of momentum, formalising the concepts of “momentum” and “discouragement effect” which have been discussed in the literature on dynamic contests.

Definition 2.1. *Let (s_A, s_B) be the state of the game with s_A and s_B being the score of A and B respectively, and p_A and p_B being the players’ probabilities to win the point.*

- *There is a momentum effect after state (s_A, s_B) if $\mu_{s_A, s_B} = p_A(s_A + 1, s_B) - p_A(s_A, s_B + 1) \geq 0$.*
- *A momentum effect is said to be larger after state (s_A, s_B) than after state (s'_A, s'_B) iff $\mu_{s_A, s_B} > \mu_{s'_A, s'_B}$.*

³Note that the state $(2,2)$ contains both the $(30,30)$ scores and the $(40,40)$ scores or “deuce” as they are strategically equivalent. For such scores, players need to win two consecutive points to win the game.

Figure 1: A tennis game between two players A and B. A player wins if he/she wins at least 4 points and two points more than the other player.

Note that we define the momentum with the winning probabilities of player A but that they can symmetrically be defined with the winning probabilities of player B since $p_A(s_A, s_B) = 1 - p_B(s_A, s_B)$. Building on the results from Konrad and Kovenock (2005) and Konrad and Kovenock (2008), we show that their finding of a momentum effect in “tug of war” and “best-of- n ” contests generalises to this type of hybrid multibattle contest.

Let’s consider two players, A and B engaged in such a dynamic contest. We model each individual sub-contest (henceforth “point”) as an all-pay auction. Let Z_A and Z_B be the prizes of winning the whole contest (henceforth “game”) for respectively players A and B . We follow Konrad and Kovenock (2008) and allow the best-of- n contest part of the game to have intermediary prizes Δ , $Z_B > \Delta \geq 0$. In the context of a tennis game this allows for the possibility that players have a preference for winning points (Sheremeta 2010).

Given the presence of a tug-of-war element at the end of the game it has a potentially infinite horizon. We assume that players maximise the expected discounted sum of per-period payoffs with $0 < \delta \leq 1$ being a common discount factor. Each player expands efforts in each state j , respectively a_j and b_j . In case of equal effort we follow Konrad and Kovenock (2005) and adopt

a tie-breaking rule advantaging the player with the largest incentives. We can then establish the existence of a momentum in such a dynamic contest:

Proposition 2.2. *If $Z_A = Z_B$, there is a momentum effect in a tennis game with the following characteristics:*

- i There is a momentum effect after all the states where the scoreline is equal or differ by only one point.*
- ii For a discounting factor not too small ($\delta > 1/2$), the momentum effect is larger after states with symmetric scorelines: $(0,0)$, $(1,1)$, $(2,2)$ than after states with asymmetric scorelines.*
- iii For states with symmetric scorelines, the momentum effect is larger towards the end of the game: $\mu_{2,2} \geq \mu_{1,1} \geq \mu_{0,0}$.*

Proposition 2.2 (proofs in the appendix) not only says that there should be a strategic momentum in a tennis game between players with similar incentives, it also indicates that the momentum should not be identical after every state. Specifically, it should be stronger after symmetric states and it should be larger at the end of the game, such as in the state $(2,2)$, than at the start, such as in the state $(0,0)$. These are testable predictions which are useful to assess whether a momentum exists and if so whether it presents these patterns, suggesting that it is a comes from players' equilibrium strategies.

While players may have similar incentives at the start of the match, each game will have a different value for each player depending on the scoreline in a match. It is therefore meaningful to consider how the result of Proposition 2.2 extends to the case of asymmetric incentives.⁴ Let $z_A(s_A, s_B)$ and $z_B(s_A, s_B)$ be the prizes to win the point (s_A, s_B) for players A and B respectively.

Proposition 2.3. *If $Z_A > Z_B$:*

- i There is a momentum effect after all the states where $z_A(s_A, s_B) > \Delta$ and $z_B(s_A, s_B) > \Delta$*

⁴Another reason to look at asymmetric incentives is the possibility for players to have different costs of effort. In the all pay auction framework, different effort cost can just be represented by different contest prizes by normalising the prizes: $Z'_A = Z_A/c_A$ and $Z'_B = Z_B/c_B$. One situation where differences in effort cost is likely to arise in tennis is the service game where it is easier for servers to win points than for receivers.

ii For Z_B close enough from Z_A , these states are $(0,0)$, $(0,1)$, $(1,1)$, $(1,2)$ and $(2,2)$.

Proposition 2.3 shows that a momentum exists also in games where players have asymmetric incentives. In such games, (i) states that there is a momentum effect after every point where both players have prizes higher than just the intermediary prize Δ . Such points are tipping points which are heavily contested. The player winning these benefits afterwards from an asymmetry in incentives in his/her favour while the losing player faces the opposite situation of a disadvantageous asymmetry in incentives. For Z_B not too far from Z_A (ii) states that these points are those where the scoreline is symmetric or those where the players with the strongest incentives trails by one point. For low values of Z_B relative to Z_A symmetric scorelines stop being tipping points and the existence of a momentum effect gets concentrated after states where A is trailing.

3 Identification strategy and Data

3.1 Identification strategy

A fundamental difficulty in the identification of a momentum is that a given state in a dynamic contest is reached as a consequence of the opposition of contestants whose characteristics are never fully observed. Any unobserved difference in ability will not only influence future performance, it will also have influenced past performance and therefore lead to different positions in the contest. There is therefore an endogeneity problem when trying to regress performance on relative positions in contest in order to estimate a momentum effects. Typically, the observation that a contestant leading in a competition has a higher level of performance than a trailing contestant does not prove the existence of a momentum as the leading contestant is most likely to have a higher level of ability.

If the contestants' abilities were fixed over time, the endogeneity problem could be addressed using a fixed effect estimator to control for the contestants unobserved characteristics. However, in most cases the hypothesis of time invariant ability is unrealistic. It is not just the case over large time span (days/months), but also within a given competition taking place in a short amount of time (few hours). In the example of a tennis match, a player may get worse if he gets a minor injury (strain, blisters) or get tired

from the physical effort (muscle soreness, cramping), he/she may get better if he/she learns how to use the weaknesses of his opponent. Assuming wrongly that the contestants' ability is fixed will bias the estimated effect of previous performance on current performance. For example, if a contestant's ability is affected by random shocks over time and follows a moving average, assuming a fixed ability will induce a positive serial auto-correlation in the errors relative to the average ability. This, in turn, will create an illusion of "momentum", with above average performance more likely to be followed by above average performance.

We propose here a new empirical strategy which solves the identification problem in the estimation of momentum effect. To identify the effect of agents' position in a dynamic contest on their current performance, one would ideally like to have an experimental setting where agents are randomly allocated to different possible states in the contest. To approximate this ideal in the field, we look for a quasi-experimental situation. In sport matches, scorelines often evolve differently around some threshold of performance. We use this to look at situations where contestants with very similar performance end up in a different relative position to each other (ahead vs behind). Looking at tennis point-by-point data, we use the fact that the probability for a player wins a point varies discontinuously as a function of the location of the ball on the court. When the ball is in, the player who hits the ball has a positive probability of winning the point. When the ball is out, the rules state that the player loses the point. Under the identification assumption that, for balls hit close enough to the line, there is no differences in average ability between players putting the ball slightly in and those putting the ball slightly out, the in/out position of the ball provides an exogenous variation in the probability to win the current point. We can use this variation to estimate the causal effect of winning the current point on winning the next point, using a fuzzy regression discontinuity design (Hahn, Todd, and Van der Klaauw 2001, Imbens and Lemieux 2008).

Formally, let's consider the study of the causal effect of a binary variable y_1 on variable y_2 , the sharp RD design approach exploits a discontinuity in the values of the variable y_1 around a threshold c of a *forcing variable* d to study the causal effect of y_1 on y_2 . The fuzzy RD design extends this approach to situations where there is a discontinuity in the expectation $E(y_1|d)$ around c . In our setting, the forcing variable d of the regression discontinuity is the relative distance of the ball bounce from the court's lines. The value of d is positive if the ball is inside the court and negative if the ball lands outside

the court. Our goal is to estimate the causal effect of winning the present point (binary variable y_1) on the chance to win the next point (variable y_2). Around the threshold $d = 0$, tennis rules imply a jump in the probability to win the current point for the player who hit the ball. This jump can be used to estimate the effect of winning the present point on the probability to win the next point, using the Wald estimator:

$$\tau_{FRD} = \frac{\lim_{d \downarrow 0} E(\Delta y_2 | d) - \lim_{d \uparrow 0} E(\Delta y_2 | d)}{\lim_{d \downarrow 0} E(\Delta y_1 | d) - \lim_{d \uparrow 0} E(\Delta y_1 | d)} = \frac{\tau_{SRD, y_2}}{\tau_{SRD, y_1}} \quad (1)$$

As the effect of the ball being in and out is not fully deterministic, but instead produces a change in the probability to win the point, the fuzzy regression design estimator rescales the exogenous variation in the probability to win the next point to provide the full causal effect of winning one point.

3.2 Data

Our dataset corresponds to the official Hawk Eye data, for all the matches played at the international professional level where this technology was used, between March 2005 and March 2009. Hawk-Eye is a computer system used in tennis and other sports to record the trajectory of the ball. Most of the matches are either from Grand slam and ATP (Association of Tennis Players) tournament or lower level ITF (International Tennis Federation) tournaments: Challengers, Futures and Satellite. In addition, some matches are from diverse cups like the Davis Cup or the Olympic games and some exhibition tournaments. Overall the dataset contains 3,561 different matches played in single.

For each point we know the position of every bounces recorded by the Hawk-Eye, as well as the player serving, the current score, and the winner of the point. The Hawk-Eye estimates very precisely the location of ball bounces with a mean prediction error of 0.36 cm.⁵ In total, we are observing the location of 1,383,522 (226,272 points) for male players and 706,105 ball bounces (108,541 points) for female players. While bounce data is automatically recorded by the tracking system, the scores, the identity of the players and the name of the server in each game are manually entered. This leads to some discrepancies, due to data entry errors. We excluded every game where we observed some discrepancy in at least one point (23.79% of the

⁵As advertised by the Hawk-Eye Innovations website: <http://www.hawkeyeinnovations.co.uk/page/sports-officiating/tennis>

ball bounces). We also excluded serves in our analysis. Serves are highly rehearsed shots where players have a high degree of control on the ball’s position and speed when hit which may allow good players to place the balls close to the lines (21.55% of the bounces are serves). Furthermore, for the main results on the existence of momentum for the next point, we did not include game points in order to ensure that the server and receiver do not change at the next point (27.42% of the bounces). We also did not include points from tie-breaks, where the scoring rule differs (4.8% of the data). We otherwise include all points, including points where players “challenged” the ruling of the line judge about the position of the ball. In such cases, the challenge of the player may lead to the point being replayed. As the challenge is motivated by the visual perception from the player about the location of the ball’s bounce mark, excluding them could lead to a selection bias around the court lines.⁶

Table 3.2 presents a description of the type of matches included in the dataset. Matches from both genders and from a wide range of competitions are included in the dataset. As the Hawk-Eye system is usually restricted to the main tournaments, the dataset contains a large proportion of matches from top tournaments (i.e. Grand Slams). Within tournaments, matches are more likely to feature top players as the system is used on the main courts and is often absent from minor courts. This aspect implies that the matches contained in the dataset are more likely to feature the best male and female players over that period. Any strategic effect to be found is therefore unlikely to be due to a lack of experience from the players.

For each bounce, the dataset records the location of a bounce mark which is an oval shape made of 51 different points. We use this bounce mark to compute the exact distance between the ball impact and the court’s lines. We then use this distance as a running variable in a regression discontinuity as the probability to win the point changes markedly around the line: within tennis rules the players hitting the ball has a positive probability to win the point if the ball is just in but has null probability to win the point if the ball is just out.

The Figure 2 shows the selected bounces for our preferred window of 4 cm. It represents a tennis court with each bounce mark in our dataset

⁶Challenge points can lead to the point being awarded to one player or to be replayed if the umpire’s initial call is overturned. We actually do not observe in the dataset which points are challenged. We included all the replayed points to ensure that we do not exclude challenged points (0.78% of the data).

Gender		Female	Male	All
Surface	Carpet	35	174	209
	Clay	125	367	492
	Grass	96	204	300
	Hard	913	1249	2162
Best of	3	1169	1469	2638
	5	0	525	525
Event ⁷	Davis Cup (<i>Fed Cup</i>)	8	18	26
	Grand Slam	453	525	978
	Olympics	19	16	35
	ATP (<i>Premier</i>)	659	100	759
	International	-	473	473
	Master	-	826	826
	Hopman Cup	30	36	66
Total		1169	1994	3163

Table 1: Break down of the matches included in the dataset

represented by its point which is the the closest to the court lines. It is this point which is relevant to measure the relative distance d to the court lines. Figure 1 shows graphically that only a very small subset of observations is used in the estimation. From the 1,383,522 (706,105) bounces in the data set for male (female) players we only use 4,939 (2,744) of them, that is 0.36% (0.39%) of the total.

For a large subsample of the matches where this information was available we collected the ATP ranking of the players as well as the odds given by the bookmakers prior to the match. We were able to get this information for 96% of the male players and 70% of the female players in our dataset.⁸

⁸We used the freely available data from <http://www.tennis-data.co.uk/alldata.php> and used the odds from the bookmaker <http://www.bet365.com.au/en/> which are available in the dataset.

Figure 2: Tennis court lines with the bounce marks present in our overall dataset (light grey) and those used for our regression discontinuity design (black) for male players.

4 Results

4.1 Momentum effect

Figure 3 shows the probability of winning the current point depending on how far the ball bounces away from the court’s line. When the ball bounces inside the court, the probability of winning the point is positive, whilst when the ball is outside the court the probability is close to zero. Note that the probability of winning the point is not zero when the ball is just out of the court. This is due to the fact that when the ball is just out of the court but close to the line, line judges may make mistakes and rule “in” some balls which were actually out. Symmetrically, some balls which are just in are sometimes ruled “out” by line judges and this leads to a drop in probability to win the point when the ball is very close from the line.⁹ For balls right on the line there is a high probability of mistakes each way.

As a consequence, Figure 3 does not show a marked discontinuity in

⁹Note that when the ball gets close to the line the probability of winning the point tends to increase due to the fact that the ball is harder to play back when it is further away in the court. However, for balls very close to the line there is a drop induced by umpires’ mistakes.

Figure 3: Probability of winning a point as a function of the distance (in meters) from which the ball hits the court’s line for male players. Estimation by local linear regression. In order to capture the possible non linearities of the regression function around the threshold we use a small bandwidth of 0.5 cm.

zero. Umpires’ mistakes may have a detrimental effects for our analysis: it blunts the discontinuity in zero which weakens the power of the fuzzy regression discontinuity estimation. To address this issue we implement a donut regression design whereby the observations in a small neighborhood of the threshold are excluded (Barreca, Lindo, and Waddell 2011, Barreca, Guldi, Lindo, and Waddell 2011, Lindo, Siminski, and Yerokhin 2013). This allows us to keep observations very close to the line while not including the balls right on the line where there are too many umpires’ mistakes for the ball position to create differences in probabilities of winning the current point. We chose a donut size of 1 cm for the results presented in the main body of the paper (all the results for donut sizes of 0 cm and 0.5 cm are included in Appendix). We also use as a non parametric estimator of the regression function a kernel regression with rectangle kernels.¹⁰

¹⁰This specification presents the interest to estimate τ_{SRD,y_1} and τ_{SRD,y_2} as local averages from point located very close from the line, rather than to extrapolate their values in zero where τ_{SRD,y_1} tends toward zero. When the derivative of the regression function is not zero, kernel regression estimators have a boundary bias for too large a bandwidth around the threshold. Given our large number of observations, we are able to use bandwidths which are very small around the threshold which mitigates this concern. Furthermore, in our case, the conditional expectation to win the next point is flat around the threshold. Our estimation of the raw effect τ_{SRD,y_2} of placing the ball just in rather than just out are therefore unbiased. Given the non linearity of the regression function depicted in Figure 3,

To ensure that our identification assumption is respected, we focus on the points where the ball fell very close to the line. Our preferred definition of “very close” which we use in the text to comment results is a window of 4 cm or less from the courts line (we explain this choice in Section 4.2). To give an order of comparison, tennis balls are designed with a diameter of approximately 6.7 cm. When they hit the court at high speed they make an oval shaped bounce mark. The biggest distance between antipodal points on such oval bounce marks is on average 9 cm in our dataset. As a comparison, the court’s lines are 5 cm wide. A distance of 4 cm is therefore roughly equal to half the size of the mark made by the bounce of the ball on the court and it is smaller than the width of the court line. Given that regression discontinuity estimates are often sensitive to bandwidth choice we follow the recommendation from Imbens and Kalyanaraman (2012) to present results with different bandwidths. We present our results in graphical form presenting point estimates and confidence intervals for a wide range of bandwidths for the distance to the line: from 2 cm to 6 cm.¹¹

Figure 4 shows the discontinuity τ_{SRD,y_2} around the threshold 0. This estimator does not reflect the effect of winning a whole point, it reflects the effect of the jump in probability to win a point which is observed in Figure 3 around the court’s lines. The difference is significant and equal to 4.16% (p=0.013, N=3,388) for males. It is not significant for female players 0.76% (p=0.739, N=1,953). The lower panels present the point estimate of τ_{SRD,y_2} for a different bandwidths. These estimates are significant at 5% for a wide range of bandwidths. This suggests that putting a ball just in rather than just out has in itself a positive effect on the probability of winning the next point for male players. The existence of a significant difference is

a boundary bias may affect our estimation of τ_{SRD,y_1} such that our estimate of the effect of a full point τ_{FRD} may be slightly biased towards 0. As a consequence our results about the effect of a full point are actually conservative.

¹¹Note that our choice of a privileged 4 cm window to discuss our results differs from the bandwidth proposed by Imbens and Kalyanaraman (2012) as a “reference point for assessing sensitivity to bandwidth choice in RD setting”, given its optimality properties. In our case, Imbens and Kalyanaraman (2012)’s suggestions give different bandwidths over different subsamples. We chose here to focus on a unique bandwidth to comment estimates across different samples. We chose a very small bandwidth to ensure that the identification assumption is valid. In order to assess the sensitivity of our RD estimator to the bandwidth choice, we present our results for a wide range of bandwidths. For completeness, we give the results in the Appendix for the Imbens and Kalyanaraman (2012)’s bandwidth over each sample.

important here as it indicates that there is a relation between an exogenous variation in the probability to win a point and the chance to win the next point as predicted by the momentum effect. The existence of a significant effect τ_{SRD,y_2} is therefore the primary indication that there is a momentum effect.

The fuzzy regression discontinuity estimator (1) rescales the difference τ_{SRD,y_2} to estimate the full effect of winning one point on the chances of winning the next point, τ_{FRD} . Looking at male players, this effect is equal to 7.21 percentage points ($p=0.012$, $N=3,388$) for balls landing within 4 cm from the court lines. Figure 5 presents the results for bandwidths between 2 cm and 6 cm. The point estimate is most of the time significant and located between 5 and 10 percentage points. Conversely, looking at female players, there is no bandwidth for which the effect is significant.¹² In the light of the literature on gender differences in behaviour in competitive environments (Niederle and Vesterlund 2011), our result suggest the men and women may react differently to incentives in a dynamic contest.

¹²While the point estimates are very close from zero, they present more variations than males' estimates when changing the size of the donut (see Appendix for different donut choices). In no specification however do we find effects close in magnitude to the one for male player significant (nor are they significant).

Figure 4: Raw effect of the ball position on the probability to win the next point for male players (left) and female players (right). The top panels present the winning probability around the threshold estimated by kernel regression with rectangle kernel of bandwidth 4 cm and a donut of 1 cm. The jump in zero is the estimation of the effect τ_{SRD,y_2} . The bottom panels present the values of this effect estimated for bandwidths ranging from 2 cm to 6 cm. The point on the curve represents the estimate of the jump visible in the upper panel.

Figure 5: Effect of winning a point on the probability of winning the next one for different bandwidths using a local Wald estimator for male players (left) a female players (right).

4.2 Robustness checks

Our identification assumption is that players who put the ball very close from the line are of similar ability on average whether the ball landed just in or just out. A possible confounding explanation of the existence of a momentum effect for male players would be that the players putting the ball slightly in have a higher ability than players putting the ball slightly out. To test whether our identification assumption is respected we run a series of robustness checks.

First, we test for the existence of a discontinuity in the density of the running variable (Imbens and Lemieux 2008). A discontinuity in the density of the balls' distance to the court lines would naturally arise if good players were able to precisely aim on the inside of the court lines. We use the test proposed by McCrary (2008) to control for such a possibility. This test consists in running a local linear regression in the values of a thinly binned histogram on each side of the threshold and to estimate the discontinuity at the threshold. Figure 17 shows an absence of manipulation of the running variable. In this figure the bandwidth is set to 4 cm and the binwidth is chosen optimally by the algorithm ($b = 0.0017$) the point estimate is 0.053 ($p = 0.33$, $N = 6,353$). In practice it means that it does not look as if good male players are able to put their ball significantly more often inside than outside when they hit a ball close to the line.

Figure 6: McCrary test of a break in the density of the ball bounces around the threshold for a bandwidth of 4 cm for male players.

Second, following the recommendation from Imbens and Lemieux (2008), we test for the existence of discontinuities in other covariates which could

have an influence on the result. We use the information we have, for a large subsample of observations, about players' professional rankings (Association of Tennis Players rankings) and about their ex-ante winning odds for the match. Using these variables, it is possible to test whether players putting the ball very close to the line but inside the court tend to have on average better ranking and better betting odds than those putting the ball just outside.

Following Klaassen and Magnus (2001) we do not use directly the ATP ranking, because the quality difference between two top ranked players (*e.g* ranked 1 and 2) is greater than between two lower ranked players (*e.g* ranked 100 and 101). Hence, we use a smoother measure of ranking proposed by Klaassen and Magnus (2001) by transforming the ATP ranking of each player into a variable R as follows:

$$R = 8 - \log_2(RANK_{ATP})$$

The betting odds, give the equivalent winning probabilities p estimated ex-ante by the betting market. Numerous studies have found that they are very good predictors of the winning probability (Williams 2009).

Figure 16 shows the estimate of the difference in ranking measure R and ex-ante winning probability p around the threshold $d = 0$ for a wide range of bandwidths. The point estimate for a bandwidth of 4 cm is 0.52 ($p = 0.921$, $N = 6139$) for differences in rankings and 0.23% ($p = 0.74$, $N = 6188$) for the ex-ante probability to win the match. This absence of discontinuity in the ability of players for a ball landing close to the line between one that put the ball in and one that put the ball out further supports our identification hypothesis.

The most convincing robustness test in our mind is however our third test. If ever players putting the ball just inside the court were to be better than those putting the ball just outside, then we should observe differences in results not only in later points but also *in previous points*. On the contrary, the momentum effect can occur only for future points. We can therefore run a placebo regression, where hitting the ball in or out is used as a predictor of the previous point. Figure 8 displays the results for the different bandwidths¹³. For a bandwidth of 4 cm the point estimate is 0.75% ($p=0.629$, $N=4,147$) which clearly shows that our assumption holds for such a bandwidth. The

¹³Since the server and receiver are switching at the end of each game the game-points were not included in the previous estimations, similarly the first point of the games are not taken into account in this computations.

Figure 7: Differences in measure of ranking (left) and ex-ante probability of winning the match (right) between players putting a ball just inside or just outside the court when considering windows between 2 cm and 6 cm around the court’s lines.

bandwidth of 4 cm is the largest one for which visually the point estimate of the placebo regression is almost zero, this is what motivated our choice of this bandwidth as a benchmark in the discussions.

Overall, these results suggest that balls landing close to the line offer an ideal quasi-experimental setting as they provide variations in winning probabilities which are not correlated with players’ abilities.

4.3 Momentum as a function of the relative position of players in the contest

Propositions 2.2 and 2.3 predict that the strategic momentum should be higher for symmetric scorelines whenever there is not too large an amount of asymmetry in incentives between players. Table 2 shows the effect of winning a point on the next one depending on the scoreline. Interestingly, the momentum effect changes with the score for male players. The effect is much stronger when the scoreline is symmetric, at (15-15) and (30-30). When comparing all the symmetric scoreline vs asymmetric scoreline, there is a significant difference for male players: the momentum effect is 10.57 ($p=0.011$, $N=1,747$), for symmetric scorelines and only 3.83 ($p=0.338$, $N=1,841$), for non symmetric scorelines. Moreover, as predicted by Proposition 2.2, the effect is highest for points at the end of the game. The momentum effect is

Figure 8: Placebo test of the effect of winning a point on the probability of winning the previous one for different bandwidths using a kernel regression and a rectangle kernel (right) and a local Wald estimator (left) for male players.

14.09 ($p=0.044$, $N=561$) for scorelines of (30-30) and 20.41 ($p=0.017$, $N=394$) for scorelines of (15-15) versus only 2.46 ($p=0.706$, $N=792$) for scorelines of (0-0). The difference between the effects for scorelines (15-15) and (30-30) is not significant.

No such patterns can be found for females. The momentum effect is not significant for any scoreline and the point estimate is actually smaller for symmetric scorelines (-0.25 , $p=0.96$, $N=1,010$) than for asymmetric ones (3.18 , $p=0.564$, $N=943$). Furthermore, there is no indication of a higher level of momentum for later points in the game with the momentum in (30-30) being almost zero (2.2 , $p=0.786$, $N=395$).

These results by scoreline support the suggestion that male players react to the changes of incentives during a dynamic contest in a way compatible with what contest theory would predict. The momentum effect is observed for states where the contest is most balanced and is higher towards the end of the game. On the contrary we do not observe such patterns for female players. This adds to the initial absence of significant overall momentum effect for female players and it may point to gender differences in how players react to incentives in dynamic contests.

Male players				Female players			
Player 1	Player 2			Player 1	Player 2		
	0	15	30		0	15	30
30	5.37 (0.502) (420)	0.02 (0.998) (601)	14.09* (0.044) (561)	30	6.61 (0.564) (215)	14.95 (0.131) (320)	2.2 (0.786) (395)
15	5.32 (0.386) (820)	20.41* (0.017) (394)		15	-8.4 (0.303) (408)	-0.74 (0.945) (220)	
0	2.46 (0.706) (792)			0	-2.78 (0.73) (395)		
Symmetric scorelines		10.57* (0.011) (1,747)		Symmetric scorelines		-0.25 (0.96) (1,010)	
Asymmetric scorelines		3.83 (0.338) (1,841)		Asymmetric scorelines		3.18 (0.564) (943)	

Table 2: Wald estimator (in percents) of the effect of winning a point on the probability of winning the next one depending on the scoreline (with a bandwidth of 5 cm and a donut of 0.5 cm). The scores 30-30 and 40-40 are merged, since they are strategically equivalent in the game of tennis (see Figure 1). p-values and sample size indicated below each coefficient. * indicates significance at the 5% level.

5 Conclusion

Given the difficulty of studying agents' dynamic behaviour in the field, our study sheds a new light on how well trained agents behave in a well defined dynamic contest. Using a large and precise dataset tracking the play of the ball in professional tennis matches, we have studied whether players react to changes in incentives which occur in a tennis contest as predicted by economic theory. We find evidence of a momentum effect for male players: winning a point in a game increase their chance to win the next one. Importantly, we are able to test predictions about the specific patterns of this momentum

effect as a function of the state in the dynamic contest. Our findings are well in line with the theory which predicts the momentum effect to be stronger when players are in a close contest and when they are toward the final states of the contest. These results suggest therefore that the momentum we observed is a consequence of the variations in incentives which occur in a dynamic contest.

However, we do not find evidence of such an effect for female players. This may suggest that the momentum effect may differ between genders. Previous research on gender differences in behaviour in competitive environments has established that males tend to have a greater preference for competitive environments (Niederle and Vesterlund 2007) and that they also tend to perform better than females in mixed tournaments (Gneezy, Niederle, and Rustichini 2003). Our results suggest that men may also adopt more efficient strategies in dynamic contests by modulating their effort as a function of the importance of the stakes present at any given moment in time. As a large part of gender inequalities arise in competitive settings such as promotion tournaments, this possible gender difference in strategic behaviour would be worth investigating further.

Note that the evidence we present in favour of a strategic momentum does not necessarily mean that male players' behaviour is optimal. There is widespread evidence in empirical research of overdispersion in contest whereby contenders invest more resources (time, effort, money) in contests than what equilibrium strategies would require to. In the case of a dynamic contest, such an overdispersion can appear if players continue to expand a lot of effort even when they don't have much chances any more to win the contest. The quote from Novak Djokovic in the introduction may suggest that while male professional players react to changes in incentives in a tennis game, they may still fail to do so optimally by expanding too much efforts on minor points.

Another question of interest is whether the strategic considerations are more or less important when teams are involved in a dynamic contest rather than single individuals. Teams decision making present additional challenges. Optimal strategies may be harder to achieve if players have to coordinate when several equilibrium strategies exist. Conversely, teams may have a greater ability to use either the knowledge of their best members or the "wisdom of the crowd" to approximate equilibrium strategies.

Finally, the well established evidence of individuals' concerns for relative positions transforms many non agonistic situations in "contest for status".

This potentially extends the relevance of contest theory well beyond formal contests within and between economic organisations. Contests for status may play a significant role both in the area of consumption choice (Hopkins and Kornienko 2004, Kuhn, Kooreman, Soetevent, and Kapteyn 2011) and for working decisions within organisations (Moldovanu, Sela, and Shi 2007, Besley and Ghatak 2008). Such decisions take place over time with agents adapting their consumption and work choices as a function of their relative standing. Understanding how agents do react to changes in relative positions in dynamic contests has therefore the potential to be of interest for a wide range of applications in economics.

References

- ABRAMITZKY, R., L. EINAV, S. KOLKOWITZ, AND R. MILL (2012): “On the optimality of line call challenges in professional tennis,” *International Economic Review*, 53(3), 939–964.
- BAR-ELI, M., S. AVUGOS, AND M. RAAB (2006): “Twenty years of hot hand research: Review and critique,” *Psychology of Sport & Exercise*, 7(6), 525–553.
- BARRECA, A., M. GULDI, J. M. LINDO, AND G. WADDELL (2011): “Saving Babies? Revisiting the effect of very low birth weight classification,” *The Quarterly Journal of Economics*, 126(4), 2117–2123.
- BARRECA, A. I., J. M. LINDO, AND G. R. WADDELL (2011): “Heaping-induced bias in regression-discontinuity designs,” Discussion paper, National Bureau of Economic Research.
- BERGER, J., AND D. POPE (2011): “Can Losing Lead to Winning?,” *Management Science*, 57(5), 817–827.
- BESLEY, T., AND M. GHATAK (2008): “Status incentives,” *The American Economic Review*, pp. 206–211.
- BREITMOSER, Y., J. H. TAN, AND D. J. ZIZZO (2010): “Understanding perpetual R&D races,” *Economic Theory*, 44(3), 445–467.
- BRYANT, J. (2011): *Game-Set-Match: A Tennis Guide*. Cengage Learning.

- CARPENTER, J., P. H. MATTHEWS, AND J. SCHIRM (2010): "Tournaments and office politics: Evidence from a real effort experiment," *The American Economic Review*, 100(1), 504–517.
- CHIAPPORI, P., S. LEVITT, AND T. GROSECLOSE (2002): "Testing Mixed-Strategy Equilibria When Players Are Heterogeneous: The Case of Penalty Kicks in Soccer," *American Economic Review*, 92(4), 1138–1151.
- DECHENAUX, E., D. KOVENOCK, AND R. SHEREMETA (forthcoming): "A survey of experimental research on contests, all-pay auctions and tournaments," *Experimental Economics*.
- DORSEY-PALMATEER, R., AND G. SMITH (2004): "Bowlers' hot hands," *The American Statistician*, 58(1), 38–45.
- FERRALL, C., AND A. SMITH (1999): "A Sequential Game Model of Sports Championship Series: Theory and Estimation," *Review of Economics and Statistics*, 81(4), 704–719.
- FERRALL, C., AND A. A. SMITH JR (1999): "A sequential game model of sports championship series: theory and estimation," *Review of Economics and Statistics*, 81(4), 704–719.
- FU, Q., C. KE, AND F. TAN (2013): "\" Success breeds success\" or \" Pride goes before a fall\"? Teams and Individuals in Best-of-Three Contests,\" Discussion paper, No. tax-mpg-rps-2013-06.
- GENAKOS, C., AND M. PAGLIERO (2012): "Interim Rank, Risk Taking, and Performance in Dynamic Tournaments," *Journal of Political Economy*, 120(4), 782–813.
- GILOVICH, T., R. VALLONE, AND A. TVERSKY (1985): "The hot hand in basketball: On the misperception of random sequences," *Cognitive Psychology*, 17(3), 295–314.
- GNEEZY, U., M. NIEDERLE, AND A. RUSTICHINI (2003): "Performance in competitive environments: Gender differences," *QUARTERLY JOURNAL OF ECONOMICS-CAMBRIDGE MASSACHUSETTS*-, 118(3), 1049–1074.

- HAHN, J., P. TODD, AND W. VAN DER KLAUW (2001): “Identification and estimation of treatment effects with a regression-discontinuity design,” *Econometrica*, 69(1), 201–209.
- HARRIS, C., AND J. VICKERS (1985): “Perfect Equilibrium in a Model of a Race,” *The Review of Economic Studies*, pp. 193–209.
- (1987): “Racing with uncertainty,” *The Review of Economic Studies*, pp. 1–21.
- HILLMAN, A. L., AND J. G. RILEY (1989): “Politically Contestable Rents and Transfers*,” *Economics & Politics*, 1(1), 17–39.
- HOPKINS, E., AND T. KORNIENKO (2004): “Running to keep in the same place: consumer choice as a game of status,” *American Economic Review*, pp. 1085–1107.
- HSU, S., C. HUANG, AND C. TANG (2007): “Minimax Play at Wimbledon: Comment,” *American Economic Review*, 97(1), 517–523.
- IMBENS, G., AND K. KALYANARAMAN (2012): “Optimal bandwidth choice for the regression discontinuity estimator,” *The Review of Economic Studies*, 79(3), 933–959.
- IMBENS, G., AND T. LEMIEUX (2008): “Regression discontinuity designs: A guide to practice,” *Journal of Econometrics*, 142(2), 615–635.
- KLAASSEN, F., AND J. MAGNUS (2001): “Are Points in Tennis Independent and Identically Distributed? Evidence From a Dynamic Binary Panel Data Model,” *Journal of the American Statistical Association*, 96(454), 500–509.
- KLAASSEN, F. J., AND J. R. MAGNUS (2009): “The efficiency of top agents: An analysis through service strategy in tennis,” *Journal of Econometrics*, 148, 72–85.
- KLUMPP, T., AND M. POLBORN (2006): “Primaries and the New Hampshire Effect,” *Journal of Public Economics*, 90(6-7), 1073–1114.
- KONRAD, K., AND D. KOVENOCK (2008): “Multi-battle contests,” *Games and Economic Behavior*.

- KONRAD, K. A. (2009): “Strategy and dynamics in contests,” *OUP Catalogue*.
- KONRAD, K. A., AND D. KOVENOCK (2005): “Equilibrium and Efficiency in the Tug-of-War,” Discussion paper, CESifo working papers.
- KUHN, P., P. KOOREMAN, A. SOETEVENT, AND A. KAPTEYN (2011): “The effects of lottery prizes on winners and their neighbors: Evidence from the Dutch Postcode Lottery,” *The American Economic Review*, pp. 2226–2247.
- LAZEAR, E., AND S. ROSEN (1981): “Rank-Order Tournaments as Optimum Labor Contracts,” *The Journal of Political Economy*, 89(5), 841.
- LINDO, J., P. SIMINSKI, AND O. YEROKHIN (2013): “Breaking the link between legal access to alcohol and motor vehicle accidents: evidence from New South Wales,” .
- MAGO, S. D., R. M. SHEREMETA, AND A. YATES (2012): “Best-of-three contest experiments: Strategic versus psychological momentum,” *International Journal of Industrial Organization*.
- MALUEG, D. A., AND A. J. YATES (2010): “Testing contest theory: evidence from best-of-three tennis matches,” *The Review of Economics and Statistics*, 92(3), 689–692.
- MCCRARY, J. (2008): “Manipulation of the running variable in the regression discontinuity design: A density test,” *Journal of Econometrics*, 142, 698 – 714.
- MOLDOVANU, B., A. SELA, AND X. SHI (2007): “Contests for status,” *Journal of Political Economy*, 115(2), 338–363.
- NIEDERLE, M., AND L. VESTERLUND (2007): “Do women shy away from competition? Do men compete too much?,” *The Quarterly Journal of Economics*, 122(3), 1067–1101.
- (2011): “Gender and competition,” *Annu. Rev. Econ.*, 3(1), 601–630.
- PALACIOS-HUERTA, I. (2003): “Professionals Play Minimax,” *Review of Economic Studies*, 70(2), 395–415.

- PASERMAN, D. (2007): “Gender Differences in Performance in Competitive Environments: Field Evidence from Professional Tennis Players,” Discussion paper, CEPR Working Paper.
- POPE, D. G., AND M. E. SCHWEITZER (2011): “Is Tiger Woods loss averse? Persistent bias in the face of experience, competition, and high stakes,” *The American Economic Review*, 101(1), 129–157.
- PRENDERGAST, C. (1999): “The provision of incentives in firms,” *Journal of Economic Literature*, pp. 7–63.
- ROMER, D. (2006): “Do Firms Maximize? Evidence from Professional Football,” *Journal of Political Economy*, 114(2), 340–365.
- SHEREMETA, R. M. (2010): “Experimental comparison of multi-stage and one-stage contests,” *Games and Economic Behavior*, 68(2), 731–747.
- SIMON, W. (1971): “‘Back-to-the-Wall’ Effect?,” *Science*, 174(4011), 774–775.
- TAYLOR, B., AND J. TROGDON (2002): “Losing to Win: Tournament Incentives in the National Basketball Association,” *Journal of Labor Economics*, 20(1), 23–41.
- TONG, K.-K., AND K. LEUNG (2002): “Tournament as a motivational strategy: Extension to dynamic situations with uncertain duration,” *Journal of Economic Psychology*, 23(3), 399–420.
- VALLERAND, R. J., P. G. COLAVECCHIO, AND L. G. PELLETIER (1988): “Psychological momentum and performance inferences: A preliminary test of the antecedents-consequences psychological momentum model,” *Journal of Sport & Exercise Psychology*.
- WALKER, M., AND J. WOODERS (2001): “Minimax Play at Wimbledon,” *American Economic Review*, 91(5), 1521–1538.
- WALKER, M., J. WOODERS, AND R. AMIR (2011): “Equilibrium play in matches: Binary Markov games,” *Games and Economic Behavior*, 71, 487–502.

WARDROP, R. (1999): “Statistical Tests for the Hot-Hand in Basketball in a Controlled Setting,” Discussion paper, Working paper, University of Wisconsin.

WILLIAMS, L. V. (2009): *Information efficiency in financial and betting markets*. Cambridge University Press.

A Appendix

A.1 Proofs

Proposition 2.2. To prove Proposition 2.2, we start with the tug of war part of the tennis game and use the results from Konrad and Kovenock (2005). The tug-of-war part of the tennis game has a possible infinite horizon. However Konrad and Kovenock show that each point is played as an individual all-pay auction where the players strategies are purely determined by one of the three states they are in. There is a unique Markov perfect equilibrium in such a game characterised by the existence of “tipping states” where both contestants invest substantial amounts of effort to win. For other states around these tipping states, an asymmetry in incentives creates an advantage for one of the players. Relative to Konrad and Kovenock, we introduce the possibility of positive and intermediary prize $\Delta \geq 0$. The size of this prize has no effect on equilibrium continuation values in each state as the intermediate prizes are netted out from a higher effort level of contestants. The proof of existence and uniqueness of a Markov perfect equilibrium in the tug of war relies only on states continuation values and it can therefore be extended to the case with a positive intermediary prize. The magnitude of Δ does impact the equilibrium distribution of efforts and the respective winning probabilities in each individual points (we describe this effect below). When $Z_A = Z_B = Z$, the continuation values at the equilibrium are given by the Proposition 1 from Konrad and Kovenock (2005): $v_A(3, 2) = \delta Z$, $v_B(3, 2) = 0$; $v_A(2, 2) = 0$, $v_B(2, 2) = 0$; $v_A(2, 3) = 0$, $v_B(2, 3) = \delta Z$. The state (2,2) is the only tipping point.

Plugging these values as end values of the game of tennis, one can use the method of Konrad and Kovenock (2008) to solve the rest of the game. Relative to their model of a multibattle contest we introduce a discount factor $\delta < 1$. This changes the continuation values and winning probabilities

without changing their results qualitatively. By backward induction the continuation values are well defined and uniquely determines point prizes and equilibrium strategies at each point. With the previous result of existence and uniqueness of a Markov perfect equilibrium in the tug of war part of the tennis game, there is therefore a unique Markov perfect equilibrium in the whole game. At any point, the prizes for the players to win the point are defined as:

$$\begin{cases} z_A(i, j) = \delta v_A(i+1, j) - \delta v_A(i, j+1) + \Delta \\ z_B(i, j) = \delta v_B(i, j+1) - \delta v_B(i+1, j) + \Delta \end{cases} \quad (2)$$

And the continuation values are:

$$\begin{cases} v_A(i, j) = \max(0, z_A(i, j) - z_B(i, j)) + \delta v_A(i, j+1) \\ v_B(i, j) = \max(0, z_B(i, j) - z_A(i, j)) + \delta v_B(i+1, j) \end{cases} \quad (3)$$

At each point the outcome is determined by an all pay auction. Assume that $z_A(i, j) \geq z_B(i, j)$ (the result is symmetric if $z_B(i, j) \geq z_A(i, j)$). Following Hillman and Riley (1989), the players' winning probabilities at each points are:

$$\begin{cases} p_A(i, j) = 1 - \frac{z_B(i, j)}{2z_A(i, j)} \\ p_B(i, j) = \frac{z_B(i, j)}{2z_A(i, j)} \end{cases} \quad (4)$$

By backward induction, equations (2), (3) and (4) uniquely determine prizes, continuation values and winning probabilities for every point of the game. From the winning probabilities, the momentum effects after each point are found to be:

$$\begin{cases} \mu_{2,2} = 1 - \frac{\Delta}{\delta Z + \Delta} \\ \mu_{1,1} = 1 - \frac{\Delta}{\delta^2 Z + \Delta} \\ \mu_{0,0} = 1 - \frac{\Delta}{\delta^3 Z + \Delta} \\ \mu_{1,0} = \mu_{0,1} = 1/2 - \frac{\Delta}{2(Z(\delta^2 - \delta^3) + \Delta)} \\ \mu_{2,1} = \mu_{1,2} = 1/2 - \frac{\Delta}{2(Z(\delta - \delta^2) + \Delta)} \\ \mu_{2,0} = \mu_{0,2} = \frac{\Delta}{2(\delta^2 Z + \Delta)} - \frac{\Delta}{2(Z(\delta - \delta^2) + \Delta)} \end{cases}$$

For $\delta \in [0, 1]$, the momentum effects $\mu_{2,2}$, $\mu_{1,1}$, $\mu_{0,0}$, $\mu_{1,0}$, $\mu_{2,1}$ are positive. The momentum effect $\mu_{2,0}$ is the only one not to always be positive. $\mu_{2,0} > 0$ iff $\delta < 1/2$. This proves point (i) of Proposition 2.2.

Figure 9: Momentum effects are larger for symmetric scorelines for δ large enough.

It is also the case that $\mu_{2,2} > \mu_{1,1} > \mu_{0,0}$ which proves point (iii) of Proposition 2.2.

In addition $\mu_{2,1} > \mu_{1,0}$. Given the previous results, the momentum effect for symmetric scorelines will be larger than for the scorelines with one point difference whenever $\mu_{0,0} > \mu_{2,1}$. Let's rewrite $\Delta = \lambda Z$. Solving the inequality $\mu_{0,0} - \mu_{2,1}$ for $(\delta, \lambda) \in [0, 1]^2$ gives for each λ a $\delta^*(\lambda)$ such that $\mu_{0,0} \geq \mu_{2,1}$ iff $\delta \geq \delta^*(\lambda)$ and $\mu_{0,0} < \mu_{2,1}$ otherwise. The Figure 9 shows the plot of the function $\delta^*(\lambda)$. For $\delta > 1/2$ the momentum effect is always larger for symmetric scorelines. This proves the point (ii) of Proposition 1. $\square$

Proposition 2.3. We start by proving (i). Let $\Sigma(k)$ be the set of possible scoreline after k points have been played in the game $\Sigma(k) = \{(s_A, s_B) : s_A > 0, s_B > 0, s_A + s_B = k\}$ ¹⁴. We say that a state $(i_k, k - i_k)$ of $\Sigma(k)$ is a *tipping state* if and only if $z_A(i_k, k - i_k) > \Delta$ and $z_B(i_k, k - i_k) > \Delta$.

At these tipping states, a momentum effect appears in the next state

¹⁴We use the same notation as Konrad and Kovenock (2008) however, for simplicity in our framework, we define here k as the number of individual contests already played while they defined as the number of contests left to play.

between winners and losers. Given that $z_A(i_k, k - i_k) > \Delta$ and $z_B(i_k, k - i_k) > \Delta$, it means that $v_A(i_k + 1, k - i_k) > v_A(i_k, k - i_k + 1)$ and $v_B(i_k, k - i_k + 1) > v_B(i_k + 1, k - i_k)$. Furthermore, by definition of continuation values, we have either $v_A(i_k + 1, k - i_k) > 0$ and $v_B(i_k + 1, k - i_k) = 0$ or $v_A(i_k + 1, k - i_k) = 0$ and $v_B(i_k + 1, k - i_k) > 0$. The same applies for state $(i_k, k - i_k + 1)$. As a consequence $v_A(i_k + 1, k - i_k)$ and $v_A(i_k, k - i_k)$ can not be jointly positive as it would contradict the inequality $v_B(i_k, k - i_k + 1) > v_B(i_k + 1, k - i_k)$. Then cannot be jointly equal to zero either as it would contradict the inequality $v_A(i_k + 1, k - i_k) > v_A(i_k, k - i_k + 1)$. Therefore it can only be the case that $v_A(i_k + 1, k - i_k) > 0$, $v_B(i_k + 1, k - i_k) = 0$, $v_B(i_k, k - i_k + 1) > 0$, $v_A(i_k, k - i_k + 1) = 0$ which implies $z_A(i_k + 1, k - i_k) > z_B(i_k + 1, k - i_k)$ and $z_B(i_k, k - i_k + 1) > z_A(i_k, k - i_k + 1)$. By definition of the equilibrium winning probabilities, this implies $\mu_{(i_k, k - i_k)} = p_A(i_k + 1, k - i_k) - p_A(i_k, k - i_k + 1) > 0$.

We now prove (ii). Let's consider the situation where $Z_B < Z_A$, with Z_B close enough from Z_A such that $Z_B > \frac{\delta + \delta^2 - \delta^3}{\delta - \delta^3 + 1} Z_A$. This condition implies $\delta Z_B > \delta^3 Z_A$. From, Konrad and Kovenock (2005)'s Proposition 1 gives the equilibrium continuation values, in that case in (3,2), (2,2) and (2,3).¹⁵

$$\begin{cases} v_A(4, 2) = Z_A \\ v_A(3, 2) = \delta Z_A \\ v_A(2, 2) = \frac{1}{1 - \delta^2}(\delta^2 Z_A - \delta^3 Z_A) \\ v_A(2, 3) = v_A(2, 4) = 0 \end{cases}$$

$$\begin{cases} v_B(4, 2) = v_B(3, 2) = v_B(2, 2) = 0 \\ v_B(2, 3) = \frac{1}{1 - \delta^2}(\delta Z_B - \delta^3 Z_A) \\ v_B(2, 4) = Z_B \end{cases}$$

From these continuation values we can compute the prize in the states of $\Sigma(k)$, (1,3) (2,2) (3,1).

¹⁵ Using the notation of Konrad and Kovenock (2005)'s $m=4$ and $j_0 = 3$.

$$\left\{ \begin{array}{l} z_A(3,1) = \delta v_A(4,1) - \delta v_A(3,2) + \Delta = \delta Z_A - \delta^2 Z_A + \Delta > \Delta \\ z_B(3,1) = \delta v_B(3,2) - \delta v_B(4,1) + \Delta = \Delta \\ \\ z_A(2,2) = \delta v_A(3,2) - \delta v_A(2,3) + \Delta = \delta^2 Z_A + \Delta > \Delta \\ z_B(2,2) = \delta v_B(2,3) - \delta v_B(3,2) + \Delta = \delta \frac{\delta Z_B - \delta^3 Z_A}{1 - \delta^2} + \Delta > \Delta \\ \\ z_A(1,3) = \delta v_A(2,3) - \delta v_A(1,4) + \Delta = \Delta \\ z_B(1,3) = \delta v_B(1,4) - \delta v_B(2,3) + \Delta = \delta Z_B - \delta \frac{\delta Z_B - \delta^3 Z_A}{1 - \delta^2} Z_A + \Delta > \Delta \end{array} \right.$$

In that case, there is therefore a unique tipping state $(i_4, 4 - i_4)$ in $\Sigma(4)$ such that $z_A(i_4, 4 - i_4) > \Delta$ and $z_B(i_4, 4 - i_4) > \Delta$. This state is (2,2) with $i_4 = 2$.

Using these prizes and continuation values, one can calculate the prizes for states in $\Sigma(3)$. Doing so, one finds a unique tipping state in (1,2) with prizes: $z_A(1,2) = \delta \frac{\delta^2 Z_A - \delta^2 Z_B}{1 - \delta^2} + \Delta$ and $z_B(1,2) = \delta^2 Z_B + \Delta$. Pursuing the backward induction process, one finds that for $Z_B > \frac{\delta Z_A}{\delta - \delta^2 + 1}$, then (1,1) is a tipping point in $\Sigma(2)$. This inequality is respected for the initial condition we have imposed on Z_B . From (1,1), by backward induction, $Z_A > Z_B$ implies that (0,1) is a tipping point in $\Sigma(1)$. Finally, for Z_B close enough from Z_A , with $Z_B > \frac{\delta + \delta^2 - \delta^3}{\delta - \delta^3 + 1} Z_A$ (the condition we posed initially) one finds by backward induction that (0,0) is also a tipping point. Figure 10 shows the range of $\lambda = Z_B/Z_A$ satisfying this condition for different values of δ . When Z_B is close enough from Z_A such that this condition is respected, the states (2,2), (1,2), (1,1), (0,1) and (0,0) are tipping states. This proves the point (ii) of the Proposition.

□

Note that for lower values of Z_B relative to Z_A , tipping points (and the associated momentum) would still exist in the game, they would simply tend to move to more asymmetric states where player A is trailing player B.

Figure 10: Condition on Z_B for Proposition 2.3

A.2 Results

A.2.1 Effect on the next point

Figure 11: Effect of winning a point on the probability of winning the next one for a donut of 0 cm.

Figure 12: Effect of winning a point on the probability of winning the next one for a donut of 0.5 cm.

A.2.2 Robustness checks

Previous point

Figure 13: Effect of winning a point on the probability of winning the previous one for a donut of 0 cm.

Figure 14: Effect of winning a point on the probability of winning the previous one for a donut of 0.5 cm.

Figure 15: Effect of winning a point on the probability of winning the previous one for a donut of 1 cm.

Figure 16: Ex ante probability of winning the match (right) and measure of ranking (left) depending on the distance from which the player hits the courts line for female players.

Figure 17: McCrary test for a bandwidth of 4 cm for female players.